# DERET FOURIER

(Jean Baptiste Joseph Fourier ahli matematika dan fisika Prancis)

- Fungsi dengan periode T = 2π (T = 2π /ω, ω = 1)

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

Koefisien deret Fourier :

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad n = 1, 2, \cdots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad n = 1, 2, \cdots$$

# Bentuk lain dalam penulisan deret Fourier

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

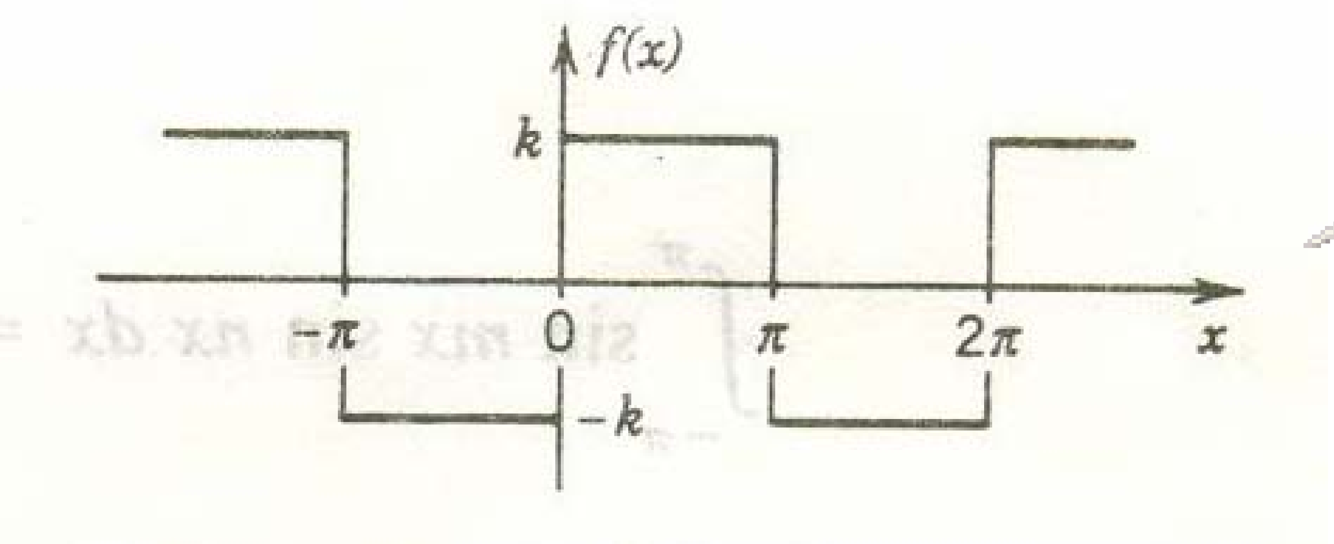
Koefisien deret Fourier :

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx \qquad n = 1,2,\cdots$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx \qquad n = 1,2,\cdots$$

Contoh :

$$f(x) = \begin{cases} -k & jika \quad -\pi < x < 0 \\ k & jika \quad 0 < x < \pi \end{cases} \qquad dan \quad f(x+2\pi) = f(x)$$




(a) The given function f(x) (Periodic square wave)

## Penyelesaian :

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{2\pi}\left[\int_{-\pi}^{0}(-k)dx + \int_{0}^{\pi} kdx\right]$$

$$= \frac{1}{2\pi}\left[-kx\Big|_{-\pi}^{0} + kx\Big|_{0}^{\pi}\right] = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-k)\cos nx dx + \int_{0}^{\pi} k\cos nx dx\right]$$

$$= \frac{1}{\pi}\left[-k\frac{\sin nx}{n}\Big|_{-\pi}^{0} + k\frac{\sin nx}{n}\Big|_{0}^{\pi}\right] = 0$$

## Penyelesaian :

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-k)\sin nx dx + \int_{0}^{\pi} k\sin nx dx\right]$$

$$= \frac{1}{\pi}\left[k\frac{\cos nx}{n}\Bigg|_{-\pi}^{0} - k\frac{\cos nx}{n}\Bigg|_{0}^{\pi}\right]$$

$$b_n = \frac{k}{n\pi}\left(\cos 0 - \cos(-n\pi) - \cos n\pi + \cos 0\right) = \frac{2k}{n\pi}\left(1 - \cos n\pi\right)$$

$$f(x) = \sum_{n=1}^{\infty}\frac{2k}{n\pi}\left(1 - \cos n\pi\right)$$

$$f(x) = \frac{4k}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x + \cdots\right)$$

# DERET FOURIER T = 2L

- Fungsi dengan periode T = 2L ( T = 2π /ω, L= π /ω)

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left( a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L} \right)$$

Koefisien deret Fourier :

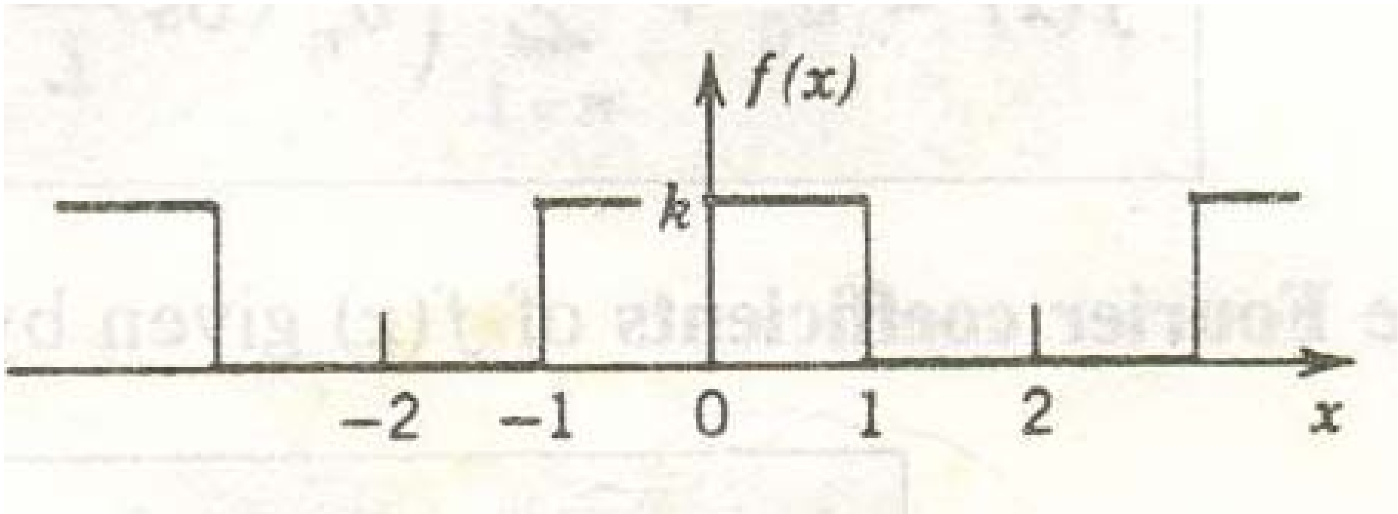
$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx \qquad n = 1,2,\cdots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx \qquad n = 1,2,\cdots$$

Contoh :

$$f(x)=\begin{cases}0 & jika \quad -2<x<-1\\ k & jika \quad -1<x<1\\ 0 & jika \quad 1<x<2\end{cases} \qquad T=2L=4,\ L=2$$




## Penyelesaian :

$$a_0 = \frac{1}{4}\int_{-2}^{2} f(x)dx = \frac{1}{4}\left[\int_{-1}^{1} kdx\right] = \frac{1}{4}\left[kx\Big|_{-1}^{1}\right] = \frac{k}{2}$$

$$a_n = \frac{1}{2}\int_{-2}^{2} f(x)\cos\frac{n\pi x}{2}dx = \frac{1}{2}\left[\int_{-1}^{1} k\cos\frac{n\pi x}{2}dx\right]$$

$$= \frac{1}{2}\left[\frac{k}{n}\sin\frac{n\pi x}{2}\Big|_{-1}^{1}\right] = \frac{2k}{n\pi}\sin\frac{n\pi}{2}$$

$$a_n = 0, \ \ jika \ \ n \ genap$$

$$a_n = \frac{2k}{n\pi}, \ jika \ \ n = 1,5,9,\cdots \qquad a_n = -\frac{2k}{n\pi}, \ jika \ \ n = 3,7,11,\cdots$$

## Penyelesaian :

$$b_n = \frac{1}{2}\int_{-2}^{2} f(x)\sin\frac{n\pi x}{2}dx = \frac{1}{2}\left[\int_{-1}^{1} k\sin\frac{n\pi x}{2}dx\right]$$

$$= \frac{1}{2}\left[\frac{k}{n}\cos\frac{n\pi x}{2}\Bigg|_{-1}^{1}\right] = 0,\ untuk\ \ n = 1,2,3,\cdots$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{2}x$$

$$f(x) = \frac{k}{2} + \frac{2k}{\pi}\left(\cos\frac{\pi}{2}x - \frac{1}{3}\cos\frac{3\pi}{2}x + \frac{1}{5}\cos\frac{5\pi}{2}x - + \cdots\right)$$