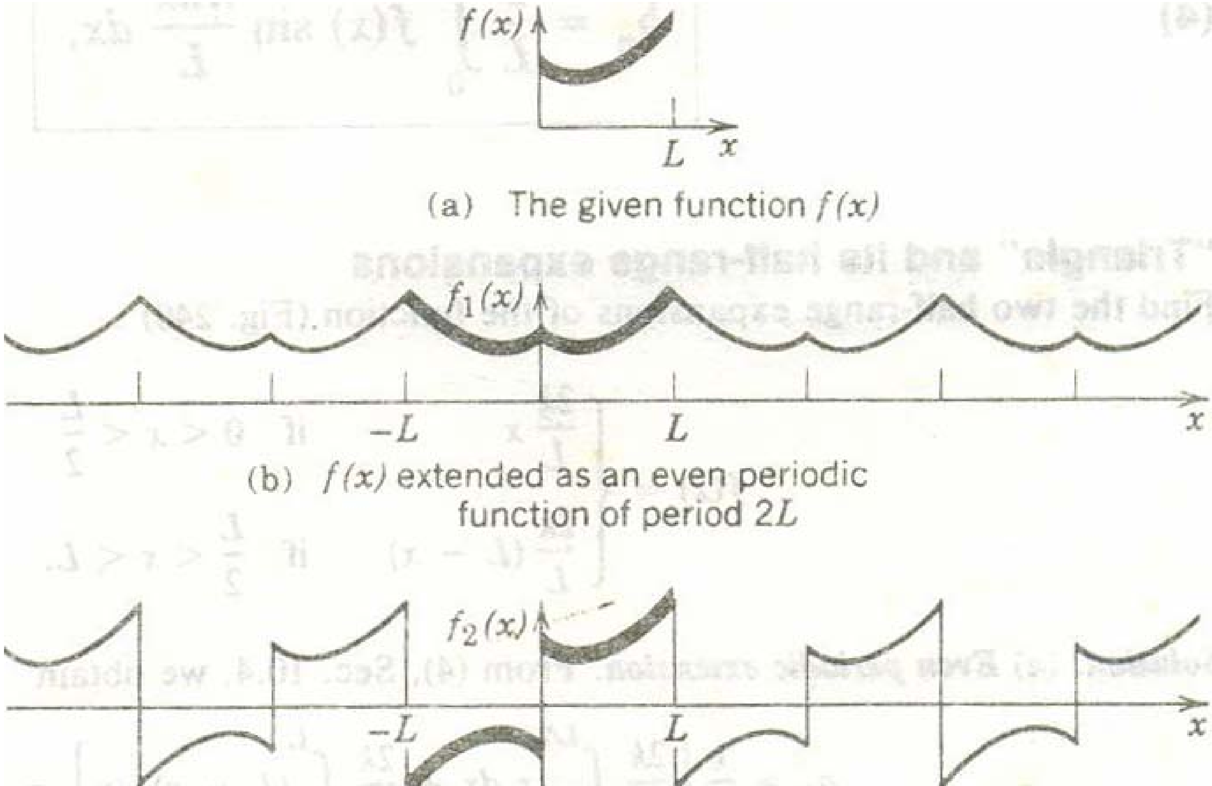
Perluasan Setengah Interval (Half-Range Expansions)



(a) The given function $f(x)$

(b) $f(x)$ extended as an even periodic function of period $2L$

(c) $f(x)$ extended as an odd periodic function of period $2L$

Perluasan Setengah Interval (Half-Range Expansions)

- Perluasan setengah interval Cosinus (Cosine Half-Range Expansions)

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{L} x$$

$$a_0 = \frac{1}{L} \int_0^L f(x)dx$$

$$a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx \qquad n = 1, 2, \cdots$$

Perluasan Setengah Interval (Half-Range Expansions)

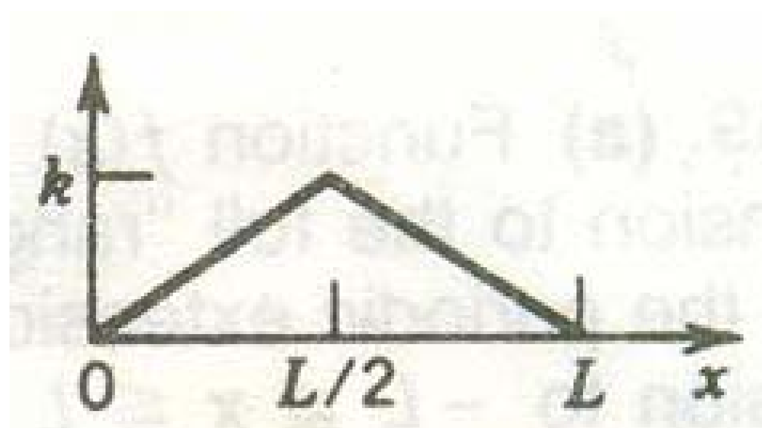
- Perluasan Setengah Interval Sinus (Sine Half-Range Expansions)

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{L} x$$

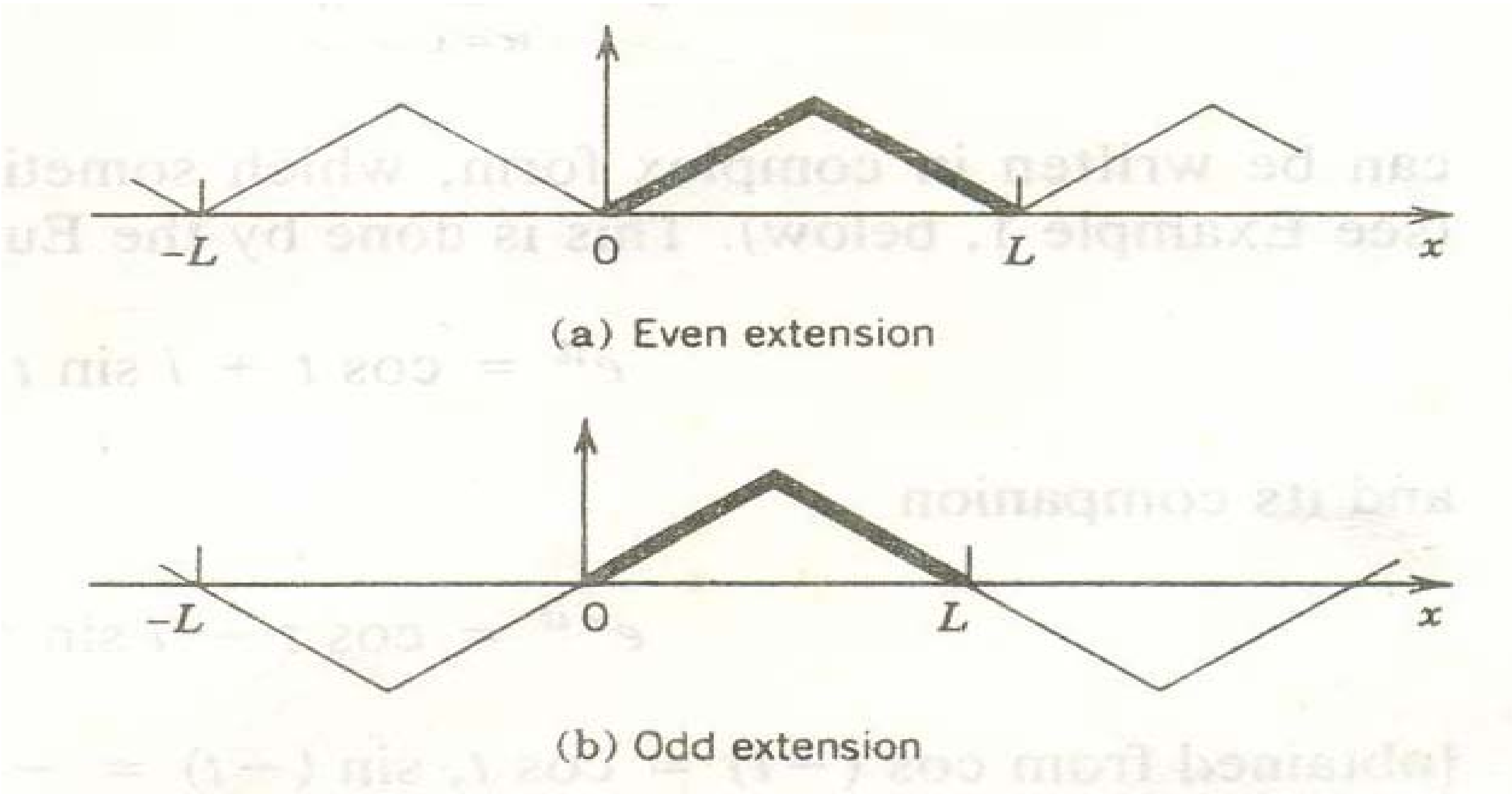
$$b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx \qquad n = 1, 2, \cdots$$

Contoh :

$$f(x) = \begin{cases} \dfrac{2k}{L}x & jika \quad 0 < x < \dfrac{L}{2} \\ \dfrac{2k}{L}(L-x) & jika \quad \dfrac{L}{2} < x < L \end{cases}$$



Penyelesaian :



Penyelesaian :

$$a_0 = \frac{1}{L}\left[\frac{2k}{L}\int_0^{L/2} x\,dx + \frac{2k}{L}\int_{L/2}^{L}(L-x)\,dx + \right] = \frac{k}{2}$$

Pertama Kedua

$$a_n = \frac{2}{L}\left[\frac{2k}{L}\int_0^{L/2} x\cos\frac{n\pi x}{L}\,dx + \frac{2k}{L}\int_{L/2}^{L}(L-x)\cos\frac{n\pi x}{L}\,dx\right]$$

Pertama $\displaystyle \int_0^{L/2} x\cos\frac{n\pi x}{L}\,dx = \frac{Lx}{n\pi}\sin\frac{n\pi}{L}x\Big|_0^{L/2} - \frac{L}{n\pi}\int_0^{L/2}\sin\frac{n\pi x}{L}\,dx$

$$= \frac{L^2}{2n\pi}\sin\frac{n\pi}{2} + \frac{L^2}{n^2\pi^2}\left(\cos\frac{n\pi}{2} - 1\right)$$

Penyelesaian :

Kedua

$$\int_{L/2}^{L}(L-x)\cos\frac{n\pi x}{L}dx=-\frac{L^2}{2n\pi}\sin\frac{n\pi}{2}-\frac{L^2}{n^2\pi^2}\left(\cos n\pi-\cos\frac{n\pi}{2}\right)$$

$$a_n=\frac{4k}{n^2\pi^2}\left(2\cos\frac{n\pi}{2}-\cos n\pi-1\right)$$

$$f(x)=a_0+\sum_{n=1}^{\infty}\frac{4k}{n^2\pi^2}\left(2\cos\frac{n\pi}{2}-\cos n\pi-1\right)\cos\frac{n\pi}{L}x$$

$$f(x)=\frac{k}{2}-\frac{16k}{\pi^2}\left(\frac{1}{2^2}\cos\frac{2\pi}{L}x+\frac{1}{6^2}\cos\frac{6\pi}{L}x+\cdots\right)$$

(Deret Cosinus Fourier)

Penyelesaian :

$$b_n = \frac{2}{L}\left[\frac{2k}{L}\int_0^{L/2} x\sin\frac{n\pi x}{L}dx + \frac{2k}{L}\int_{L/2}^{L}(L-x)\sin\frac{n\pi x}{L}dx\right]$$

$$b_n = \frac{8k}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$f(x) = \sum_{n=1}^{\infty}\left(\frac{8k}{n^2\pi^2}\sin\frac{n\pi}{2}\right)\sin\frac{n\pi}{2}x$$

$$f(x) = \frac{8k}{\pi^2}\left(\frac{1}{1^2}\sin\frac{\pi}{L}x + \frac{1}{3^2}\sin\frac{3\pi}{L}x + \frac{1}{5^2}\sin\frac{5\pi}{L}x + \cdots\right)$$

(Deret Sinus Fourier)