# Deret Fourier Komplek (Eksponensial)

- Rumus Euler

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos nx = \frac{e^{jnx} + e^{-jnx}}{2}$$

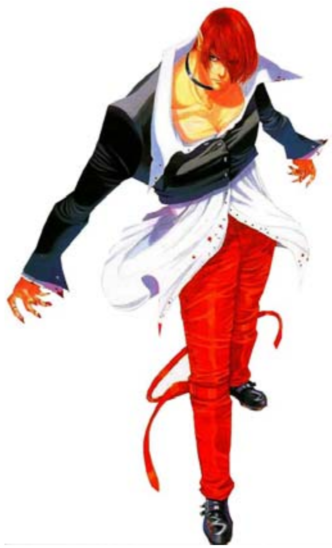
$$\sin nx = \frac{e^{jnx} - e^{-jnx}}{j2}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$$

$$\left(a_n \cos nx + b_n \sin nx\right) = \left( a_n\left(\frac{e^{jnx} + e^{-jnx}}{2}\right) + b_n\left(\frac{e^{jnx} - e^{-jnx}}{j2}\right)\right)$$

# Deret Fourier Komplek (Eksponensial)

$$\left(a_n \cos nx + b_n \sin nx\right) = \left( a_n \left( \frac{e^{jnx} + e^{-jnx}}{2} \right) + b_n \left( \frac{e^{jnx} - e^{-jnx}}{2j} \right) \right)$$



$$= \underbrace{\frac{1}{2}\left(a_n - jb_n\right)}_{c_n} e^{jnx} + \underbrace{\frac{1}{2}\left(a_n + jb_n\right)}_{k_n} e^{-jnx}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left( c_n e^{jnx} + k_n e^{-jnx} \right)$$

# Deret Fourier Komplek (Eksponensial)

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(c_n e^{jnx} + k_n e^{-jnx}\right)$$

$$c_0 = a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$c_{n=} \frac{1}{2}\left(a_n - jb_n\right) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-jnx}\,dx$$

$n$ = 1,2, . . .

$$k_{n=} \frac{1}{2}\left(a_n + jb_n\right) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{jnx}\,dx$$

# Deret Fourier Komplek (Eksponensial)

Jika $k_n = c_{-n}$

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{jnx}$$

$$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-jnx} dx \qquad n = 0, \pm 1, \pm 2, \cdots$$

Untuk T = 2L

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{jn\pi x/L} \qquad C_n = \frac{1}{2L} \int_{-L}^{L} f(x) e^{-jn\pi x/L} dx$$

Bila $T = \dfrac{2\pi}{\omega}$ maka $f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}$ $\quad C_n = \dfrac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} f(t) e^{-jn\omega t} dt$

Contoh :

$$f(t) = \begin{cases} 1 & -\frac{T}{4} < t < \frac{T}{4} \\ 0 & \text{untuk yang lain} \end{cases}$$

Penyelesaian :

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$C_n = \frac{1}{T}\int_{-T/4}^{T/4} 1 e^{-j2n\pi t/T} dt = \frac{1}{T}\left[\frac{e^{-j2n\pi t/T}}{-j2n\pi/T}\right]_{-T/4}^{T/4}$$

$$= \frac{-1}{j2n\pi/T}\left(e^{-jn\pi t/2} - e^{jn\pi t/2}\right) = \frac{1}{n\pi}\left(\frac{e^{jn\pi t/2} - e^{-jn\pi t/2}}{2j}\right) = \frac{1}{n\pi}\sin\frac{n\pi}{2}$$

$$f(t) = \sum_{n=-\infty}^{\infty}\left(\frac{1}{n\pi}\sin\frac{n\pi}{2}\right)e^{j2n\pi t/T}$$

Untuk *n = 0* hasilnya tidak valid maka harus dicari $C_0$

$$C_0 = \frac{1}{T}\int_{-T/4}^{T/4} 1\, dt = \frac{1}{T}[t]_{-T/4}^{T/4} = \frac{1}{2}$$