

# Transformasi Fourier Diskrit (Discrete Fourier Transform (d.f.t))

---

Transformasi Fourier diskrit dari barisan  $f[n]$ ,  
 $n = 0, 1, 2, \dots, N-1$  adalah :

$$F[k] = \sum_{n=0}^{N-1} f[n] e^{-2jnk\pi / N} \text{ untuk } k = 0, 1, 2, \dots, N-1$$

Invers Transformasi Fourier diskrit dari barisan  $F[k]$ ,  
 $k = 0, 1, 2, \dots, N-1$  adalah :

$$D^{-1}\{F[k]\} = f[n] = \frac{1}{N} \sum_{k=0}^{N-1} F[k] e^{2jnk\pi / N} \text{ untuk } n = 0, 1, 2, \dots, N-1$$

Contoh :

Carilah transformasi Fourier diskrit dari barisan  $f[n] = 1, 2, -5, 3$

---

Penyelesaian :

$$D\{f[n]\} = F[k] = \sum_{n=0}^{N-1} f[n] e^{-2jnk\pi/N} \text{ untuk } k = 0, 1, 2, \dots, N-1$$

$$N = 4$$

$$F[k] = \sum_{n=0}^3 f[n] e^{-2jnk\pi/4} \text{ untuk } k = 0, 1, 2, 3$$

$$k = 0$$

$$F[0] = \sum_{n=0}^3 f[n] e^0 = 1 + 2 + (-5) + 3 = 1$$

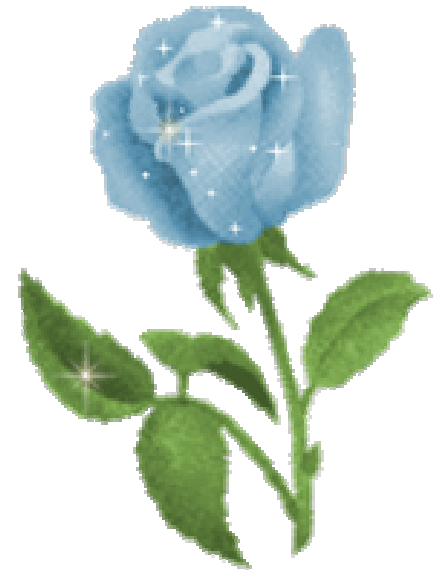
$$k = 1$$

---

$$\begin{aligned} F[1] &= \sum_{n=0}^3 f[n] e^{-2jn\pi/4} = 1 + 2e^{-2j\pi/4} + (-5)e^{-2j2\pi/4} + 3e^{-2j3\pi/4} \\ &= 1 + 2e^{-j\pi/2} + (-5)e^{-j\pi} + 3e^{-3j\pi/2} \\ &= 1 + 2(-j) - 5(-1) + 3j = 6 + j \end{aligned}$$

$$k = 2$$

$$\begin{aligned} F[2] &= \sum_{n=0}^3 f[n] e^{-2jn2\pi/4} = \sum_{n=0}^3 f[n] e^{-jn\pi} \\ &= 1 + 2e^{-j\pi} + (-5)e^{-2j\pi} + 3e^{-3j\pi} \\ &= 1 + 2(-1) - 5(1) + 3(-1) = -9 \end{aligned}$$



$$k = 3$$

---

$$\begin{aligned} F[3] &= \sum_{n=0}^3 f[n]e^{-2jn3\pi/4} = \sum_{n=0}^3 f[n]e^{-3jn\pi/2} \\ &= 1 + 2e^{-3j\pi/2} + (-5)e^{-3j\pi} + 3e^{-9j\pi/2} \\ &= 1 + 2(j) - 5(-1) + 3(-j) = 6 - j \end{aligned}$$

Jadi transformasi Fourier diskrit dari barisan 1, 2, -5, 3 adalah barisan 1, 6 + j, -9, 6 - j

Contoh :

Carilah invers transformasi Fourier diskrit dari barisan

$$F[k] = -4, 1, 0, 1$$

---

Penyelesaian :

$$D^{-1}\{F[k]\} = f[n] = \frac{1}{N} \sum_{k=0}^{N-1} F[k] e^{2jnk\pi/N} \text{ untuk } n = 0, 1, 2, \dots, N-1$$

$$N = 4$$

$$\begin{aligned} D^{-1}\{F[k]\} = f[n] &= \frac{1}{4} \sum_{k=0}^{N-1} F[k] e^{2jnk\pi/4} \quad n = 0, 1, 2, 3 \\ &= \frac{1}{4} \sum_{k=0}^{N-1} F[k] e^{jnk\pi/2} \quad n = 0, 1, 2, 3 \\ &= \frac{1}{4} (-4 + 1e^{jn\pi/2} + 0e^{jn\pi} + 1e^{3jn\pi/2}) \end{aligned}$$

$$\text{Untuk } n = 0, 1, 2, 3 \text{ maka } f[n] = -\frac{1}{2}, -1, -\frac{3}{2}, -1$$

# Representasi Transformasi Fourier Diskrit dengan Matrik

---

Transformasi Fourier diskrit dari barisan  $f[n]$ ,  
 $n = 0, 1, 2, \dots, N-1$  adalah :

$$F[k] = \sum_{n=0}^{N-1} f[n] e^{-2jnk\pi/N} \text{ untuk } k = 0, 1, 2, \dots, N-1$$

suku  $e^{-2jnk\pi/N} = \left( e^{-2j\pi/N} \right)^{nk} = W^{nk}$

$$F[k] = \sum_{n=0}^{N-1} f[n] W^{nk}$$

$$F[k] = f[0]W^0 + f[1]W^k + f[2]W^{2k} + f[3]W^{3k} + \dots + f[N-1]W^{(N-1)k}$$

$k = 0, 1, 2, \dots, N-1$

# Representasi Transformasi Fourier Diskrit dengan Matrik

---

$$F[k] = \sum_{n=0}^{N-1} f[n]W^{nk}$$

$$F[k] = f[0]W^0 + f[1]W^k + f[2]W^{2k} + f[3]W^{3k} + \dots + f[N-1]W^{(N-1)k}$$

$$k = 0, 1, 2, \dots, N-1$$

$$F[0] = f[0]W^0 + f[1]W^0 + f[2]W^0 + f[3]W^0 + \dots + f[N-1]W^0$$

$$F[1] = f[0]W^0 + f[1]W^1 + f[2]W^2 + f[3]W^3 + \dots + f[N-1]W^{(N-1)}$$

$$F[2] = f[0]W^0 + f[1]W^2 + f[2]W^4 + f[3]W^6 + \dots + f[N-1]W^{2(N-1)}$$

$$\vdots = \dots$$

$$F[N-1] = f[0]W^0 + f[1]W^{(N-1)} + f[2]W^{2(N-1)} + f[3]W^{3(N-1)} + \dots \\ + f[N-1]W^{(N-1)(N-1)}$$

# Representasi Transformasi Fourier Diskrit dengan Matrik

---

$$\begin{pmatrix} F[0] \\ F[1] \\ F[2] \\ \vdots \\ F[N-1] \end{pmatrix} = \begin{pmatrix} W^0 & W^0 & W^0 & \dots & W^0 \\ W^0 & W^1 & W^2 & \dots & W^{N-1} \\ W^0 & W^2 & W^4 & \dots & W^{2(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ W^0 & W^{N-1} & W^{2(N-1)} & \dots & W^{(N-1)(N-1)} \end{pmatrix} \begin{pmatrix} f[0] \\ f[1] \\ f[2] \\ \vdots \\ f[N-1] \end{pmatrix}$$

dengan  $W = e^{-2j\pi/N}$



Carilah representasi matrik dari transformasi Fourier diskrit

**Contoh :** dari tiga titik. Gunakan matrik tersebut untuk mencari

transformasi Fourier diskrit dari barisan  $f[n] = 4, -7, 11$

**Penyelesaian :**

$$N = 3, W = e^{-2j\pi/N}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{-2j\pi/3} & e^{-4j\pi/3} \\ 1 & e^{-4j\pi/3} & e^{-8j\pi/3} \end{pmatrix} \text{ dirubah kebentuk } \textit{Cartesian} \begin{pmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - j\frac{\sqrt{3}}{2} & -\frac{1}{2} + j\frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - j\frac{\sqrt{3}}{2} \end{pmatrix}$$

$$\begin{pmatrix} F[0] \\ F[1] \\ F[2] \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - j\frac{\sqrt{3}}{2} & -\frac{1}{2} + j\frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - j\frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 4 \\ -7 \\ 11 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 + 15,588j \\ 2 - 15,588j \end{pmatrix}$$

# Transformasi Fourier Diskrit dengan MATLAB (Fast Fourier Transform (FFT))

---



Using Toolbox Path Cache. Type "help toolbox\_path\_cache" for more info.  
To get started, select "MATLAB Help" from the Help menu.

```
>> f=[1 2 -5 3];  
>> y=fft(f)
```

y =

```
1.0000      6.0000 + 1.0000i -9.0000      6.0000 - 1.0000i
```

```
>> f=[4 -7 11];  
>> y=fft(f)
```

y =

```
8.0000      2.0000 +15.5885i  2.0000 -15.5885i
```