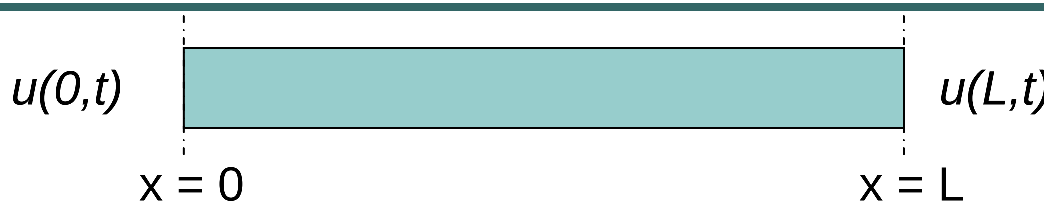
Aliran Panas Dimensi Satu



$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \qquad c^2 = \frac{K}{\sigma\rho}$$

$$K = \textit{diffusivity thermal}$$

$$\sigma = \textit{specific heat}$$

$$\rho = \textit{density material}$$

Syarat batas $u(0,t) = 0, \quad u(L,t) = 0$ untuk seluruh t

Kondisi awal temperatur $u(x,0) = f(x)$ $\quad [f(x)$ diberikan$]$

Langkah I Pemisahan Variabel

$$u(x,t) = F(x).G(t)$$

$$\frac{\partial u}{\partial t} = \left\{\frac{\partial}{\partial t}\left[F(x).G(y)\right]\right\} = F\left[\frac{\partial G}{\partial t}\right] = FG'$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left\{\frac{\partial}{\partial x}\left[F(x).G(y)\right]\right\} = \frac{\partial}{\partial x}\left[\frac{\partial F}{\partial x}\right]G = F''G$$

$$FG' = c^2 F''G \qquad \text{dibagi dengan } c^2 FG$$

$$\frac{G'}{c^2 G} = \frac{F''}{F} = k \qquad k = -p^2 \ (dipilih)$$

$$\Rightarrow F'' + p^2 F = 0$$

$$\Rightarrow G' + c^2 p^2 G = 0$$

Langkah II Penyelesaian Syarat Batas

Penyelesaian $F'' + p^2 F = 0$

$$F(x) = A\cos px + B\sin px$$

$$u(0,t) = F(0).G(t) = 0, \qquad u(L,t) = F(L).G(t) = 0$$

$$F(0) = 0, \qquad F(L) = 0$$

$$F(0) = A = 0, \qquad F(L) = B\sin pL = 0$$

$$F_n(x) = \sin\frac{n\pi}{L}x, \qquad n = 1,2,\cdots$$

Penyelesaian $G' + p^2 G = 0 \qquad p = \dfrac{n\pi}{L},$

$$G' + \lambda_n{}^2 G = 0 \qquad \lambda_n = \frac{n\pi}{L} \qquad G_n(t) = B_n e^{-\lambda_n{}^2 t}$$

$$u_n(x,t) = F_n(x).G_n(t) = B_n \sin\frac{n\pi}{L} x e^{-\lambda_n{}^2 t}$$

Langkah III Penyelesaian dengan Deret Fourier

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi}{L} x e^{-\lambda_n{}^2 t} \qquad \lambda_n = \frac{cn\pi}{L}$$

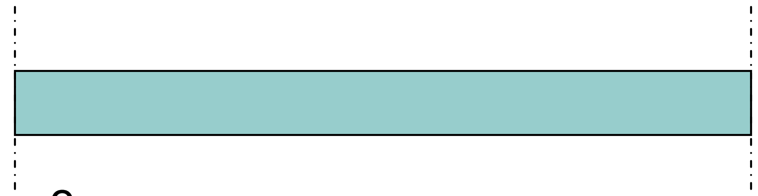
Syarat awal $u(x,0)$

$$u(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi}{L} x = f(x)$$

$$B_n = \frac{2}{L}\int_0^L f(x) \sin\frac{n\pi}{L} x dx \qquad n = 1, 2, \cdots$$

Contoh :

Carilah tempratur batang tembaga yang panjangnya 3 cm dan tempratur awal 25^0C



Penyelesaian :

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi}{L} x e^{-\lambda_n{}^2 t} \qquad \lambda_n = \frac{cn\pi}{L}$$

$$L = 3$$

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi}{3} x e^{-\lambda_n{}^2 t} \qquad \lambda_n = \frac{cn\pi}{3}$$

Syarat awal $u(x,0)$

$$u(x,0)=\sum_{n=1}^{\infty} B_n \sin\frac{n\pi}{3}x = f(x) = 25$$

$$B_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi}{L}x\,dx \qquad n=1,2,\cdots$$

$$B_n=\frac{2}{3}\int_0^3 25\sin\frac{n\pi}{3}x\,dx=\frac{50}{n\pi}\left(\cos\frac{n\pi}{3}\right)_0^3=\frac{50}{n\pi}(1-\cos n\pi)$$

$$u(x,t)=\sum_{n=1}^{\infty}\left(\frac{50}{n\pi}(1-\cos n\pi)\right)\sin\frac{n\pi}{3}xe^{-\lambda_n{}^2 t} \qquad \lambda_n=\frac{cn\pi}{3}$$

$$=\frac{100}{\pi}\left(\sin\frac{\pi}{3}xe^{-\left(c\pi/3\right)^2 t}+\frac{1}{3}\sin\pi\, xe^{-(c\pi)^2 t}+\cdots\right)$$