Faktor Integrasi

Suatu persamaan diferensial tidak eksak

$$P(x,y)dx + Q(x,y)dy = 0$$

$$F(x,y) \quad Faktor \;\; Integrasi$$

$$FPdx + FQdy = 0$$

$$Syarat\ eksak \;\; \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial}{\partial y}(FP) = \frac{\partial}{\partial x}(FQ)$$

Faktor Integrasi

Misalkan faktor integrasi F hanya tergantung satu variabel misalnya x maka :

$$F\frac{\partial P}{\partial y}=\frac{dF}{dx}Q+F\frac{\partial Q}{\partial x}$$

Dibagi dengan FQ dan menyusun kembali susku - sukunya :

$$\frac{1}{\text{F}}\frac{dF}{dx}=\frac{1}{Q}\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)$$

Contoh :

Apakah persamaan diferensial berikut eksak atau tidak

$$(4x + 3y^2)dx + 2xydy = 0$$

Faktor Integrasi

$$\frac{1}{\mathrm{F}}\frac{dF}{dx} = \frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = \frac{1}{2xy}(6y - 2y) = \frac{2}{x}$$

$$\frac{1}{\mathrm{F}}\frac{dF}{dx} = \frac{2}{x}, \qquad \ln|F| = 2\ln|x|, \qquad F(x) = x^2$$

$$x^2(4x + 3y^2)dx + x^2(2xy)dy = 0$$

$$(4x^3 + 3x^2y^2)dx + (2x^3y)dy = 0$$

$$\frac{\partial P}{\partial y} = 6x^2y \qquad \frac{\partial Q}{\partial x} = 6x^2y$$

Karena $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$, maka persamaan tersebut menjadi eksak

Persamaan Diferensial Linear

Suatu persamaan diferensial orde pertama dikatakan linear,

jika persamaan tersebut dapat dituliskan sebagai :

$$y' + p(x)y = r(x)$$

$r(x) = 0$ Homogen

$r(x) \neq 0$ Tak homogen

Penyelesaian Homogen

$$y' + p(x)y = 0$$

$$\frac{dy}{y} = -p(x)dx, \qquad \ln|y| = -\int p(x)dx + c$$

$$y(x) = Ae^{-\int p(x)dx} \qquad A = e^{c}$$

Penyelesaian tak homogen:

$$(py-r)dx+dy=0$$

$$\frac{1}{F}\frac{dF}{dx}=p(x), \qquad \ln|F|=\int p(x)dx$$

$$F(x)=e^{h(x)} \qquad \text{dengan } h(x)=\int p(x)dx$$

$$h'=p$$

$$e^h(y'+h'y)=e^h r \qquad (e^h y)'=e^h y'+e^h h' y$$

$$(e^h y)'=e^h r \qquad \text{dengan mengintegralkan didapat}$$

$$e^h y=\int e^h r dx+c \qquad \text{dibagi dengan } e^h$$

$$y(x)=e^{-h}\left[\int e^h r dx+c\right], \qquad h=\int p(x)dx$$

Contoh:

Selesaikanlah persamaan diferensial linear

$$y' - y = e^{2x}$$

Penyelesaian :

$$p = -1, \qquad r = e^{2x}, \qquad h = \int p\,dx = -x$$

$$y(x) = e^{x}\left[\int e^{-x}e^{2x}dx + c\right] = e^{x}\left[e^{x} + c\right] = ce^{x} + e^{2x}$$

Contoh :

Selesaikan $\quad xy' + y + 4 = 0$

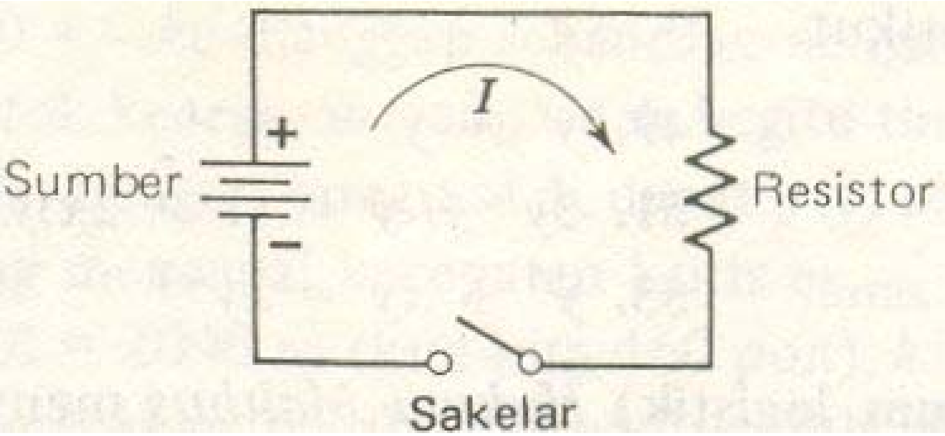
Penyelesaian :

$$y' + \frac{1}{x}y = -\frac{4}{x}, \qquad p = \frac{1}{x}, \qquad r = -\frac{4}{x},$$

$$h = \int p\,dx = \ln|x|, \qquad e^{h} = x, \qquad e^{-h} = \frac{1}{x}$$

$$y(x) = \frac{1}{x}\left[\int x\left(-\frac{4}{x}\right)dx + c\right] = \frac{c}{x} - 4$$

Pembentukan Model



$$E_R = IR$$

$$E_L = L\frac{dI}{dt}$$

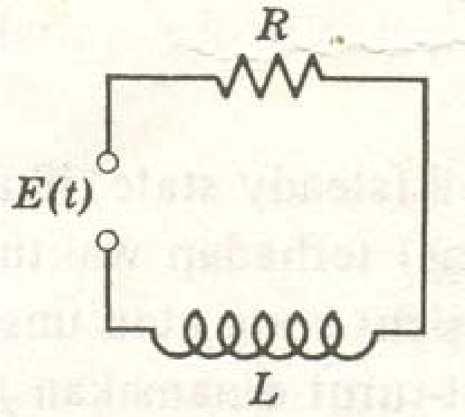
$$E_C = \frac{1}{C}Q, \qquad I(t) = \frac{dQ}{dt}$$

$$E_C = \frac{1}{C}\int I(t)dt$$

Pembentukan Model

- Rangkaian RL

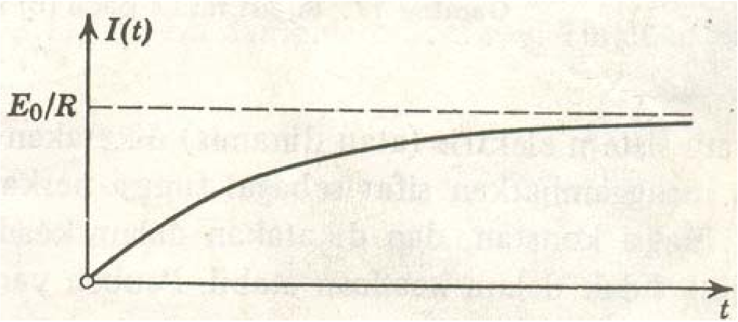
$$L\frac{dI}{dt} + RI = E(t)$$



Sumber tegangan konstan $E(t) = E_o$

$$I(t) = e^{-\alpha t}\left[\frac{E_o}{L}\int e^{\alpha t}dt + c\right] \qquad (\alpha = R/L)$$

$$= \frac{E_o}{R} + ce^{-(R/L)t}$$

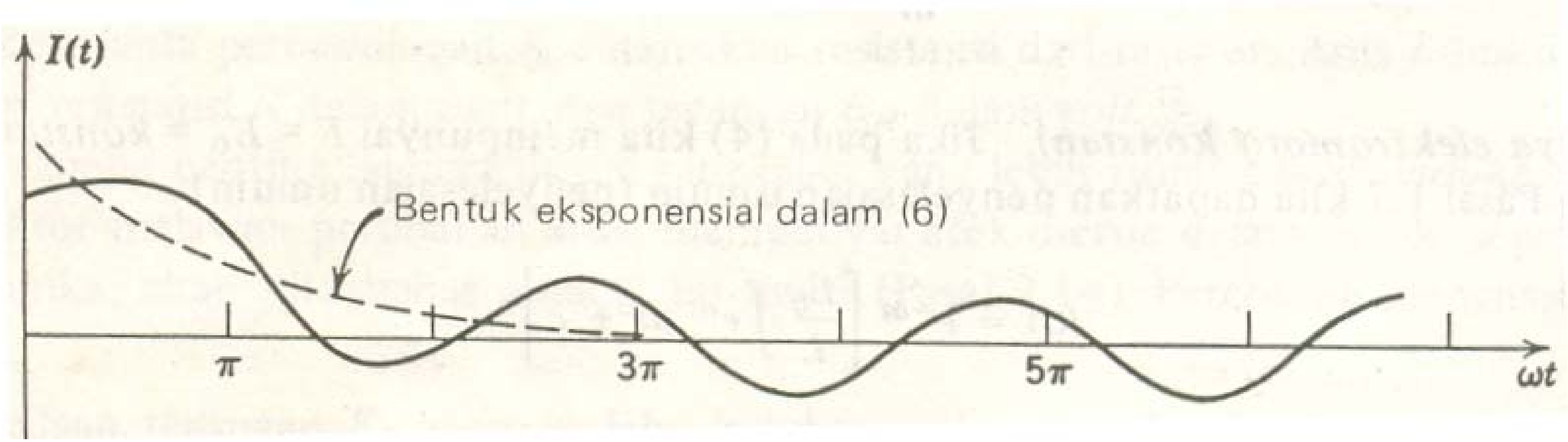


Sumber tegangan sinusiodal $E(t) = E_o \sin \omega t$

$$I(t) = e^{-\alpha t}\left[\frac{E_o}{L}\int e^{\alpha t}\sin \omega t dt + c\right] \qquad (\alpha = R/L)$$

$$= ce^{-(R/L)t} + \frac{E_o}{R^2+\omega^2 L^2}\left(R\sin\omega t - \omega L\cos\omega t\right)$$

$$I(t) = ce^{-(R/L)t} + \frac{E_o}{\sqrt{R^2+\omega^2 L^2}}\sin\left(\omega t - \delta\right) \qquad \delta = arc\tan\frac{\omega L}{R}$$



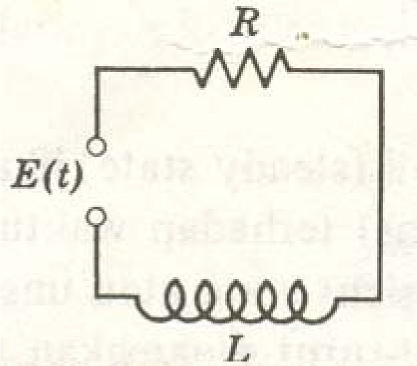
Contoh :

$$R = 10\Omega, \qquad L = 2\ henry$$

$$a.\ E(t) = 40\ V \qquad I(0) = 0$$

$$b.\ E(t) = 50\sin 5t\ V$$

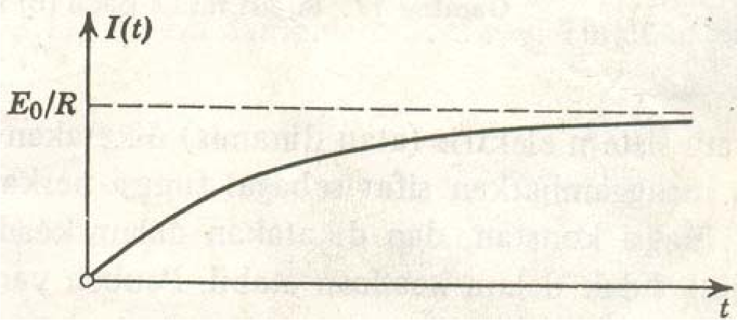
$$LI' + RI = E(t)$$



Sumber tegangan konstan $E(t) = 40$

$$I(t) = e^{-\alpha t}\left[\frac{E_o}{L}\int e^{\alpha t}dt + c\right] \qquad (\alpha = R/L)$$

$$= \frac{E_o}{R} + ce^{-(R/L)t}$$



$$I(t) = \frac{40}{10} + ce^{-5t} \qquad I(0) = \frac{40}{10} + c = 0 \qquad c = -4$$

$$I(t) = 4 - 4e^{-5t} = 4(1 + e^{-5t})$$

Sumber tegangan sinusiodal $E(t) = 50\sin 5t$

$$I(t) = e^{-\alpha t}\left[\frac{E_o}{L}\int e^{\alpha t}\sin\omega t dt + c\right] \qquad (\alpha = R/L)$$

$$= ce^{-(R/L)t} + \frac{E_o}{R^2 + \omega^2 L^2}\left(R\sin\omega t - \omega L\cos\omega t\right)$$

$$I(t) = ce^{-5t} + \frac{50}{10^2 + 5^2 2^2}\left(10\sin 5t - 10\cos\omega t\right)$$

$$= ce^{-5t} + \frac{5}{20}\left(10\sin 5t - 10\cos\omega t\right) = ce^{-5t} + \frac{5}{2}\left(\sin 5t - \cos 5t\right)$$

$$I(0) = c + \frac{5}{2} = 0 \qquad c = \frac{5}{2}$$

$$\boxed{I(t) = \frac{5}{2}\left(\sin 5t - \cos 5t\right) + \frac{5}{2}e^{-5t}}$$

Rumpun Kurva dan Trayektori Ortogonal

- Dua rumpun kurva yang saling berrpotongan tegak lurus dikatakan saling ortogonal dan membentuk jaringan ortogonal. Rumpun kurva tersebut disebut Trayektori Ortogonal

Diberikan rumpun kurva $F(x, y) = 0$

dapat digambarkan oleh persamaan diperensial

$$y' = f(x, y)$$

Persamaan diferensial dari trayektori ortogonalnya adalah :

$$y' = -\frac{1}{f(x, y)}$$

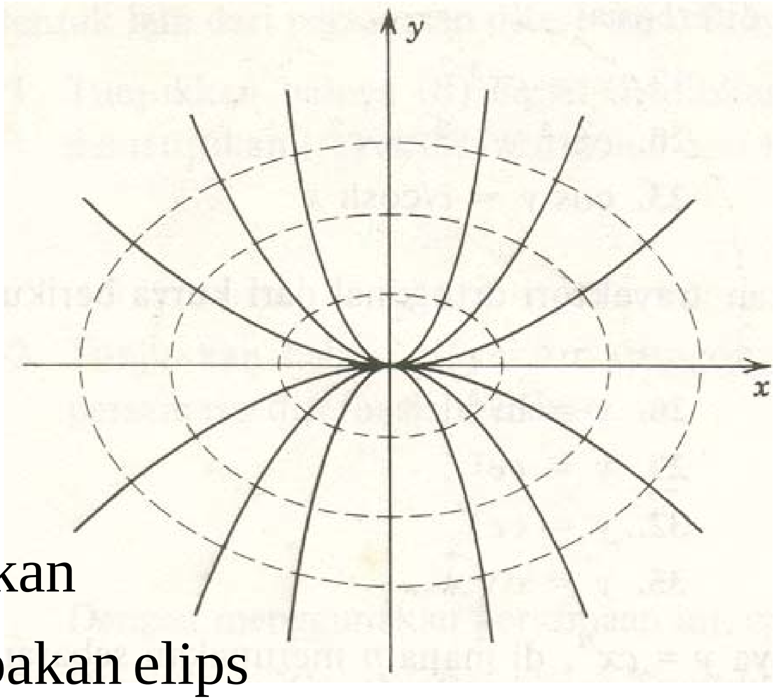
Contoh :

Persamaan diperensial dari rumpun parabol

$$y = cx^2$$

$$y' = 2cx \qquad (c = y / x^2)$$

$$y' = \frac{2y}{x}$$



Trayektori Ortogonal

$$y' = -\frac{1}{2y/x} = -\frac{x}{2y}$$

Dengan memisahkan dan mengintegralkan

diperoleh trayektori ortogonalnya merupakan elips

$$\frac{x^2}{2} + y^2 = c$$