Kombinasi Linear, Bebas Linear dan Tak Bebas Linear

Kombinasi Linear

$$c_1\mathbf{a_{(1)}} + \cdots + c_m\mathbf{a_{(m)}} \qquad (c_1, \cdots, c_2 \text{ sembarang sekalar dan } \mathbf{a_{(1)}}, \cdots, \mathbf{a_{(m)}} \text{ sembarang vektor})$$

$$c_1\mathbf{a_{(1)}} + c_2\mathbf{a_2} + \cdots + c_m\mathbf{a_{(m)}} = 0$$

Bebas Linear

Jika $c_1, \cdots, c_m$ sama dengan nol, maka persamaan menjadi $0 = 0$

Tak Bebas (bergantung) Linear

Jika $c_1, \cdots, c_m$ tida semuanya sama dengan nol,

Rank (Peringkat Matrik)

- Rank dari matrik A merupakan jumlah maksimum dari baris atau kolom A yang bebas linear, yang ekivalen dengan jumlah baris terbanyak dalam sub matrik bujur sangkar dari A yang memiliki determinan tidak sama dengan nol

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \\ 3 & 8 & 10 \end{bmatrix} \Rightarrow$$

Mempunyai rank A = 2 karena kolom terakhir merupakan kombinasi linear dari dua kolom lainnya (6 x kolom pertama dikurangi 1kali kolom kedua) yang bebas linear atau sub matrik 2 x 2 determinannya tidak sama dengan nol

$$c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 10 \end{bmatrix}$$

$$c_1 = 6 \qquad c_2 = -1$$

$$\det(\mathbf{A}) = \begin{bmatrix} 2 & 4 \\ 3 & 8 \end{bmatrix} = 4 \neq 0$$

Adjoin Matrik

$$\mathbf{A} = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{bmatrix} \quad \text{elemen kofaktornya} \quad \mathbf{C} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$A_{11} = (+1)\begin{vmatrix} 1 & 6 \\ 4 & 0 \end{vmatrix} = (+1)(0-24) = -24$$

$$A_{12} = (-1)\begin{vmatrix} 4 & 6 \\ 1 & 0 \end{vmatrix} = (-1)(0-6) = 6$$

$$A_{13} = (+1)\begin{vmatrix} 4 & 1 \\ 1 & 4 \end{vmatrix} = (+1)(16-1) = 15$$

$$A_{21} = (-1)\begin{vmatrix} 3 & 5 \\ 4 & 0 \end{vmatrix} = (-1)(0-20) = 20$$

$$A_{22} = (+1)\begin{vmatrix} 2 & 5 \\ 1 & 0 \end{vmatrix} = (+1)(0-5) = -5$$

$$A_{23} = (-1)\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = (-1)(8-3) = -5$$

$$A_{31} = (+1)\begin{vmatrix} 3 & 5 \\ 1 & 6 \end{vmatrix} = (+1)(18-5) = 13$$

$$A_{22} = (-1)\begin{vmatrix} 2 & 5 \\ 4 & 6 \end{vmatrix} = (-1)(12-20) = 8$$

$$A_{33} = (+1)\begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix} = (+1)(2-12) = -10$$

$$\mathbf{C} = \begin{bmatrix} -24 & 6 & 15 \\ 20 & -5 & -5 \\ 13 & 8 & -10 \end{bmatrix} \qquad \text{adj}\,\mathbf{A} = \mathbf{C}^{\mathbf{T}} = \begin{bmatrix} -24 & 20 & 13 \\ 6 & -5 & 8 \\ 15 & -5 & -10 \end{bmatrix}$$

Invers Matrik

$$\mathbf{A}^{-1} = \frac{\mathbf{adj\ A}}{|\mathbf{A}|} = \frac{\mathbf{C}^{\mathbf{T}}}{|\mathbf{A}|}$$

$$\mathbf{A} = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{bmatrix} \Rightarrow \mathbf{C} = \begin{bmatrix} -24 & 6 & 15 \\ 20 & -5 & -5 \\ 13 & 8 & -10 \end{bmatrix} \Rightarrow \text{adj}\,\mathbf{A} = \mathbf{C}^{\mathbf{T}} = \begin{bmatrix} -24 & 20 & 13 \\ 6 & -5 & 8 \\ 15 & -5 & -10 \end{bmatrix}$$

Determinan $$|\mathbf{A}| = \begin{vmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{vmatrix} = 2(0-24) - 3(0-6) + 5(16-1) = 45$$

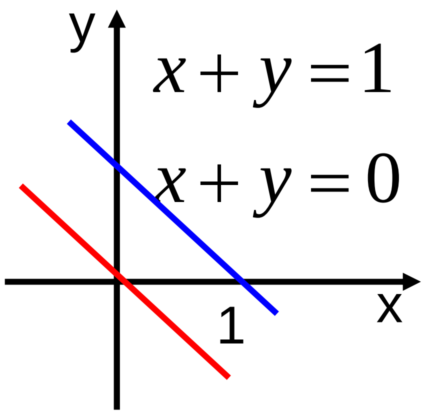
$$\mathbf{A}^{-1} = \frac{\mathbf{adj\ A}}{|\mathbf{A}|} = \frac{\mathbf{C}^{\mathbf{T}}}{|\mathbf{A}|} = \frac{1}{45}\begin{bmatrix} -24 & 20 & 13 \\ 6 & -5 & 8 \\ 15 & -5 & -10 \end{bmatrix} = \begin{bmatrix} -0.5333 & 0.4444 & 0.2889 \\ 0.1333 & -0.1111 & 0.1778 \\ 0.3333 & -0.1111 & -0.2222 \end{bmatrix}$$

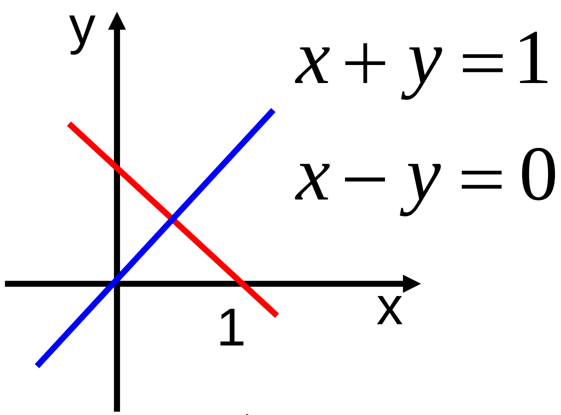
$$\mathbf{AA}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}, \quad (\mathbf{AC})^{-1} = \mathbf{C}^{-1}\mathbf{A}^{-1}, \quad \mathbf{AC}(\mathbf{AC})^{-1} = \mathbf{I}, \quad \mathbf{I} = \textbf{matrik satuan}$$

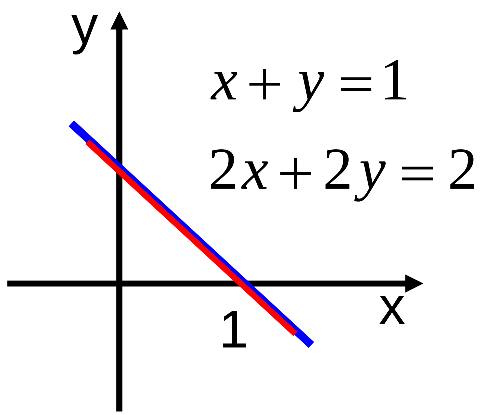
Penyelesaian Sistem Persamaan Linear

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

a. Tidak ada penyelesaian jika kedua garis itu sejajar
b. Ada tepat satu penyelesaian jika kedua garis itu berpotongan
c. Ada tak hingga banyaknya penyelesaian jika kedua garis itu berimpit







Metode Determinan (aturan Cramer)

$$x_1 = \frac{\mathbf{D_1}}{\mathbf{D}} \qquad x_2 = \frac{\mathbf{D}_2}{\mathbf{D}} \qquad x_3 = \frac{\mathbf{D}_3}{\mathbf{D}}$$

Contoh :

$$\begin{aligned} 2x_1 - x_2 + 2x_3 &= 2 \\ x_1 + 10x_2 - 3x_3 &= 5 \\ -x_1 + x_2 + x_3 &= -3 \end{aligned} \Rightarrow \begin{bmatrix} 2 & -1 & 2 \\ 1 & 10 & -3 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix}$$

$$\mathbf{D} = \begin{vmatrix} 2 & -1 & 2 \\ 1 & 10 & -3 \\ -1 & 1 & 1 \end{vmatrix} = 46 \quad \mathbf{D}_1 = \begin{vmatrix} 2 & -1 & 2 \\ 5 & 10 & -3 \\ -3 & 1 & 1 \end{vmatrix} = 92 \quad \mathbf{D}_2 = \begin{vmatrix} 2 & 2 & 2 \\ 1 & 5 & -3 \\ -1 & -3 & 1 \end{vmatrix} = 0$$

$$\mathbf{D}_3 = \begin{vmatrix} 2 & -1 & 2 \\ 1 & 10 & 5 \\ -1 & 1 & -3 \end{vmatrix} = -46 \quad x_1 = \frac{\mathbf{D_1}}{\mathbf{D}} = \frac{92}{46} = 2, \quad x_2 = \frac{\mathbf{D}_2}{\mathbf{D}} = \frac{0}{46} = 0 \quad x_3 = \frac{\mathbf{D}_3}{\mathbf{D}} = \frac{-46}{46} = -1$$

Metode Invers Matrik

$$\mathbf{A}\cdot\mathbf{x}=\mathbf{b} \qquad \mathbf{x}=\mathbf{A}^{-1}\cdot\mathbf{b}$$

$$x_1+2x_2+x_3=4$$
$$3x_1-4x_2-2x_3=2$$
$$5x_1+3x_2+5x_3=-1$$

$$\Rightarrow \begin{bmatrix}1 & 2 & 1\\ 3 & -4 & -2\\ 5 & 3 & 5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}4\\ 2\\ -1\end{bmatrix}$$

$$A=\begin{bmatrix}1 & 2 & 1\\ 3 & -4 & -2\\ 5 & 3 & 5\end{bmatrix}$$

Invers $\Rightarrow$

$$A^{-1}=-\frac{1}{35}\begin{bmatrix}-14 & -7 & 0\\ -25 & 0 & 5\\ 29 & 7 & -10\end{bmatrix}$$

$$\mathbf{x}=\mathbf{A}^{-1}\cdot\mathbf{b}=-\frac{1}{35}\begin{bmatrix}-14 & -7 & 0\\ -25 & 0 & 5\\ 29 & 7 & -10\end{bmatrix}\cdot\begin{bmatrix}4\\ 2\\ -1\end{bmatrix}=-\frac{1}{35}\cdot\begin{bmatrix}-70\\ -105\\ 140\end{bmatrix}=\begin{bmatrix}2\\ 3\\ -4\end{bmatrix}$$

$$x_1=2,\quad x_2=3,\quad x_3=-4$$