Masalah Nilai Awal

Contoh :

$$y''+y'-2y=0, \quad y(0)=4, \quad y'(0)=-5$$

Langkah 1:

$$\lambda^2+\lambda-2=0$$

$$\lambda_1=\frac{-1+\sqrt{1+8}}{2}=1 \quad \text{dan} \quad \lambda_2=\frac{-1-\sqrt{1+8}}{2}=-4$$

$$PU. \quad y=c_1e^{x}+c_2e^{-2x}$$

Langkah 2:

$$y'=c_1e^{x}-2c_2c^{-2x}$$

$$y(0)=c_1+c_2=4$$

$$y'(0)=c_1-2c_2=-5$$

$$\Rightarrow \quad c_1=1, \quad c_2=3$$

$$\boxed{PK \quad y=e^{x}+3c^{-2x}}$$

Contoh :

$$y''+2y'+5y=0, \qquad y(0)=1, \qquad y'(0)=5$$

Langkah 1:

$$\lambda^2+2\lambda+5=0$$

$$\lambda_1=\frac{-2+\sqrt{4-20}}{2}=-1+j2 \quad \text{dan} \quad \lambda_2=\frac{2-\sqrt{4-20}}{2}=-1-j2$$

$$P.U.\ \ y=e^{-x}\left(A\cos 2x+B\sin 2x\right)$$

Langkah 2:

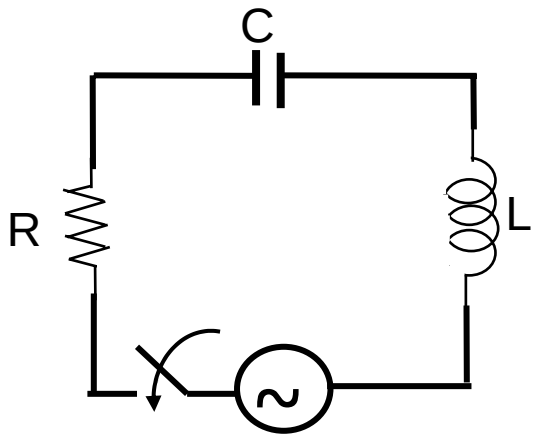
$$y(0)=A+0=1 \qquad A=1$$

$$y'=e^{-x}\left(-A\cos 2x-B\sin 2x-2A\sin 2x+2B\cos 2x\right)$$

$$y'(0)=-A+2B=-1+2B=5 \qquad B=3$$

$$\boxed{P.K.\ \ y=e^{-x}\left(\cos 2x+3\sin 2x\right)}$$

Pembentukan Model Rangkaian Listrik



$$E(t) = E_0 \sin \omega t$$

$$E_R = IR$$

$$E_L = L\frac{dI}{dt}$$

$$E_C = \frac{1}{C}Q, \qquad I(t) = \frac{dQ}{dt}$$

$$E_C = \frac{1}{C}\int I(t)dt$$

$$E_L + E_R + E_C = E(t)$$

$$LI' + RI + \frac{1}{C}\int I(t)dt = E_0 \sin \omega t$$

$$LI'' + RI' + \frac{1}{C}I = E_0 \omega \cos \omega t$$

atau

$$LQ'' + RQ' + \frac{1}{C}Q = E_0 \sin \omega t$$

$$LI''+RI'+\frac{1}{C}I = E_0\omega\cos\omega t$$

$$L\lambda^2 + R\lambda + \frac{1}{C} = 0 \qquad ay''+by'+cy = r(x)$$

$$\lambda_1 = \alpha + j\beta, \quad \lambda_2 = \alpha - j\beta$$

$$\alpha = -\frac{R}{2L}, \quad \beta = \frac{\sqrt{R^2 - 4LC}}{2L} \quad \Rightarrow \quad \boxed{I_h = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}}$$

$$I_p = K\cos\omega t + M\sin\omega t \qquad K = \frac{-E_0 S}{R^2 + S^2} \qquad M = \frac{E_0 S}{R^2 + S^2}$$

$$S = \omega L - \frac{1}{\omega C} \qquad \textit{(S = reaktansi)}$$

$$I_p = I_0\sin(\omega t - \theta) \qquad \tan\theta = -\frac{K}{M} = \frac{S}{R} \quad \Rightarrow \quad \boxed{I = I_h + I_p}$$

$$I_0 = \sqrt{K^2 + M^2} = \frac{E_0}{\sqrt{R^2 + S^2}}$$

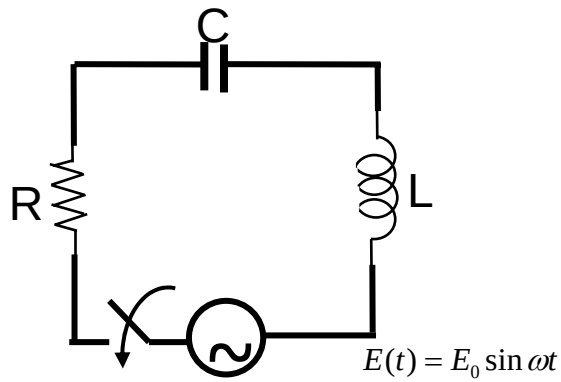
$\sqrt{K^2 + M^2}$ — impedansi

Contoh :

$$R = 100\Omega, \qquad L = 0{,}1 \; henry, \qquad C = 10^{-3} \; farad$$

$$E(t) = 155 \sin 377t \qquad I(0) = 0, \qquad Q(0) = 0$$

$$0{,}1I'' + 100I' + 1000I = 155 \cdot 377 \sin 377t$$



$$S = 37{,}7 - \frac{1}{0{,}377} = 35$$

$$I_p(t) = K \cos \omega t + M \sin \omega t$$

$$K = \frac{-155 \cdot 35}{100^2 + 35^2} = -0{,}484 \qquad M = \frac{-155 \cdot 100}{100^2 + 35^2} = 1{,}380$$

$$0{,}1\lambda^2 + 100\lambda + 1000 = 0$$

$$\lambda_1 = -10 \quad \lambda_2 = -990$$

$$I_h(t) = c_1 e^{-10t} + c_2 e^{-990t}$$

$$I(t) = c_1 e^{-10t} + c_2 e^{-990t} - 0{,}484 \cos 377t + 1{,}380 \sin 377t$$

$$I(0) = 0$$

$$I(0) = c_1 + c_2 - 0{,}484 = 0$$

$$Q(0) = 0$$

Kondisi awal
Arus *I(0) = 0*
Muatan *Q(0) = 0*

$$I'(t) = \frac{1}{L}\left[E(t) - RI(t) - \frac{1}{C}Q(t) \right]$$

$$E(0) = 0, \quad I(0) = 0, \quad Q(0) = 0, \quad \text{jadi} \quad I' = 0$$

$$I'(t) = -10c_1 e^{-10t} - 990c_2 e^{-990t} + (0{,}484)(377)\sin 377t + (1{,}380)(377)\cos 377t$$

$$I'(0) = -10c_1 - 990c_2 + (1{,}380)(377) = 0$$

$$\begin{aligned} c_1 + \quad c_2 &= 0{,}484 \\ -10c_1 - 990c_2 &= -520{,}26 \end{aligned} \quad \Rightarrow \quad \begin{aligned} c_1 &= -0{,}042 \\ c_2 &= 0{,}526 \end{aligned}$$

$$I(t) = -0{,}042e^{-10t} + 0{,}526e^{-990t} - 0{,}484\cos 377t + 1{,}380\sin 377t$$