VEKTOR (1)

IR. I NYOMAN SETIAWAN, MT
PS. TEKNIK ELEKTRO
UNIVERSITAS UDAYANA
E-mail :man_awan@yohoo.com
setiawan@ee.unud.ac.id

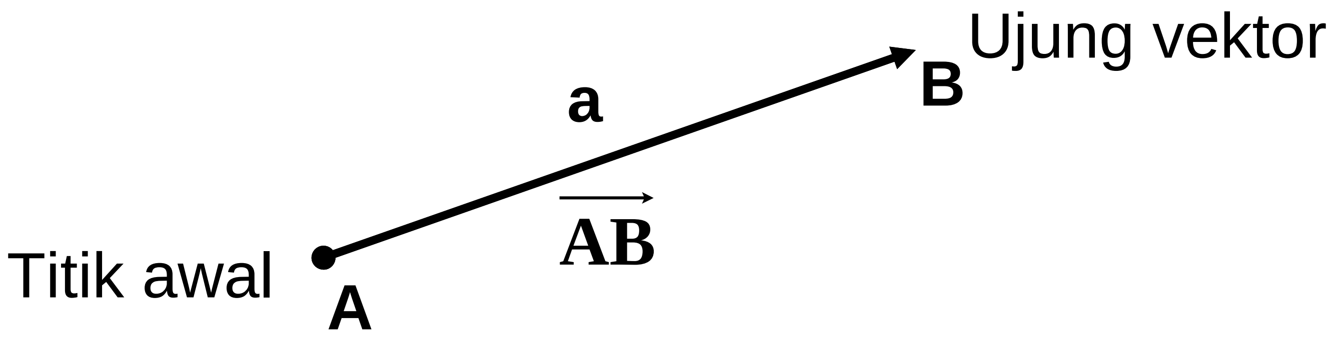
Vektor

Besaran fisis :

1. Besaran Skalar : besaran yang cukup dinyatakan oleh sebuah bilangan dengan satuannya yang sesuai
2. Besaran Vektor : besaran yang dinyatakan oleh sebuah bilangan dengan satuannya yang sesuai dan arahnya

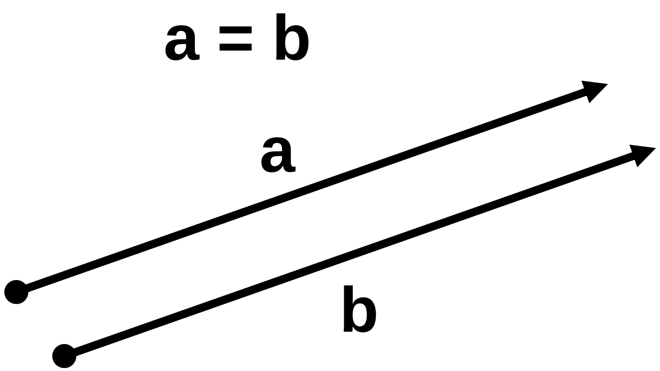
Penggambaran Vektor

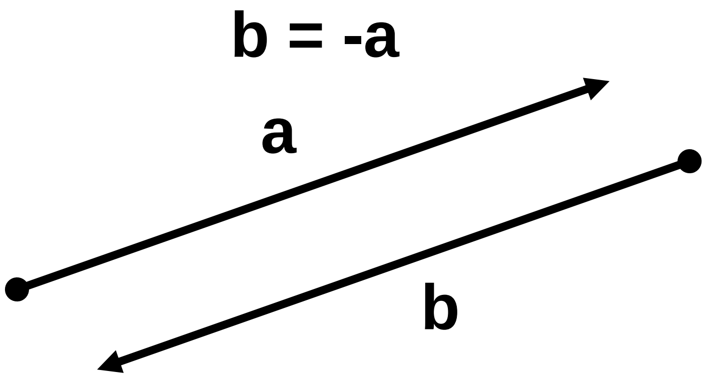
Vektor adalah suatu segmen garis berarah



Kesamaan Vektor

- Dua vektor dikatakan sama jika dan hanya jika kedua vektor itu mempunyai panjang sama dan arah sama

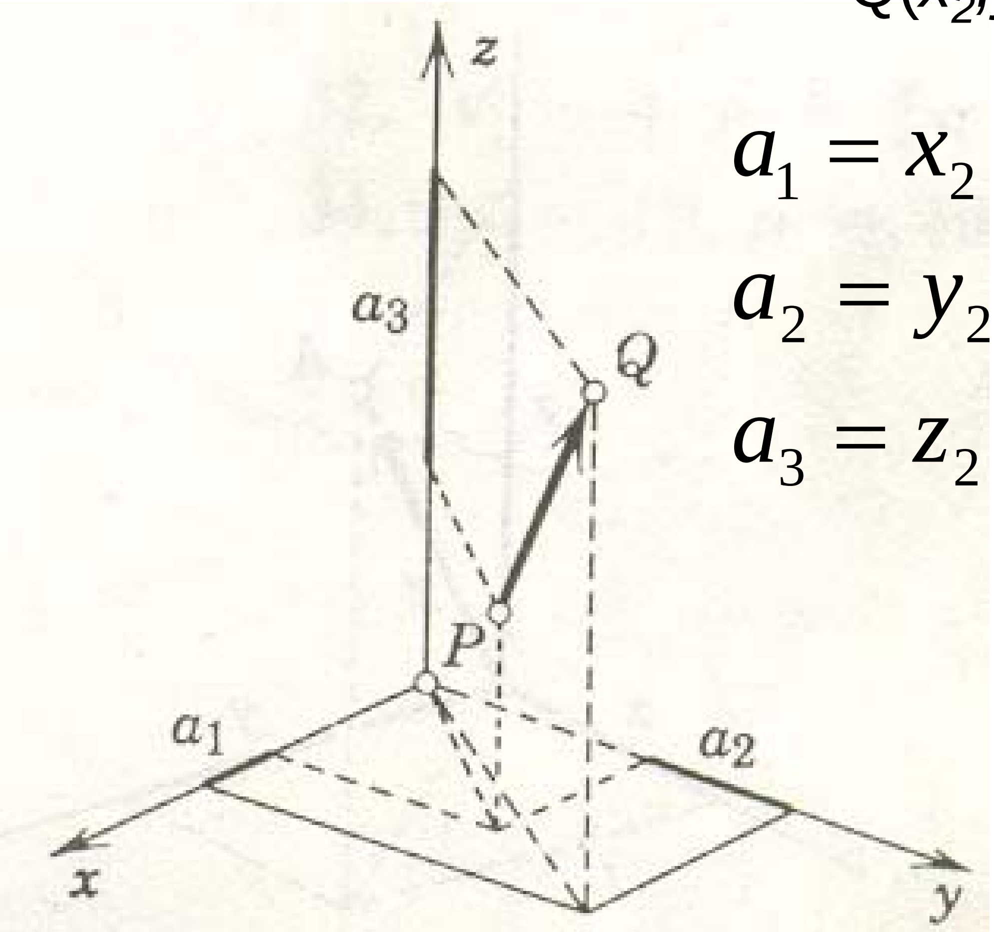




Vektor yang mempunyai panjang sama tetapi dengan arah berbeda

Komponen Vektor

P(x_1,y_1,z_1)
Q(x_2,y_2,z_2)



$$a_1 = x_2 - x_1 \qquad a_x = x_2 - x_1$$

$$a_2 = y_2 - y_1 \quad \text{atau} \quad a_y = y_2 - y_1$$

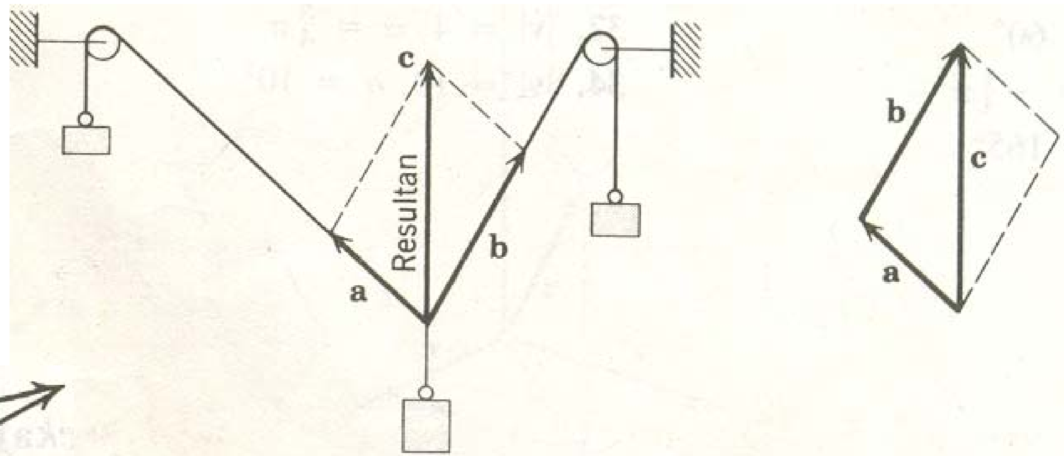
$$a_3 = z_2 - z_1 \qquad a_z = z_2 - z_1$$

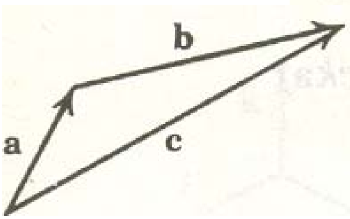
$$\mathbf{a} = [a_1, a_2, a_3]$$

Panjang vektor **a**

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

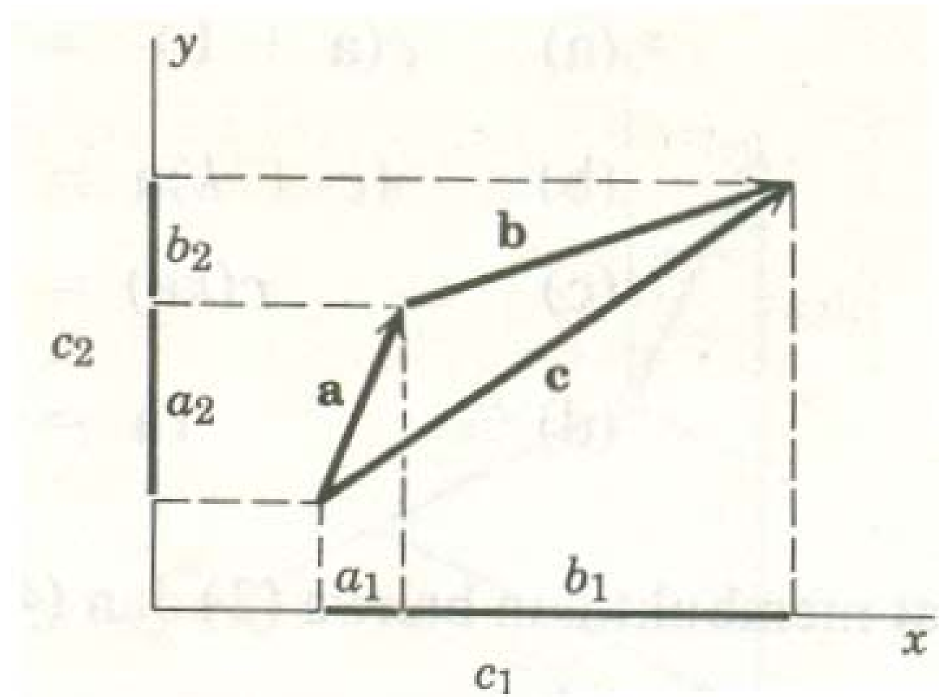
Penjumlahan Vektor





$$\mathbf{a}+\mathbf{b}=\left[a_1+b_1,\, a_2+b_2,\, a_3+b_3\right]$$

$$\mathbf{c}=\mathbf{a}+\mathbf{b}$$



Sifat-sifat

1. $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$
2. $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$
3. $\mathbf{a}+\mathbf{0}=\mathbf{0}+\mathbf{a}=\mathbf{a}$
4. $\mathbf{a}+(-\mathbf{a})=\mathbf{0}$

Perkalian Vektor dengan Skalar

Jika $\mathbf{a} = [a_1, a_2, a_3]$ dan c bilangan, maka

$$c\mathbf{a} = [ca_1, ca_2, ca_3]$$

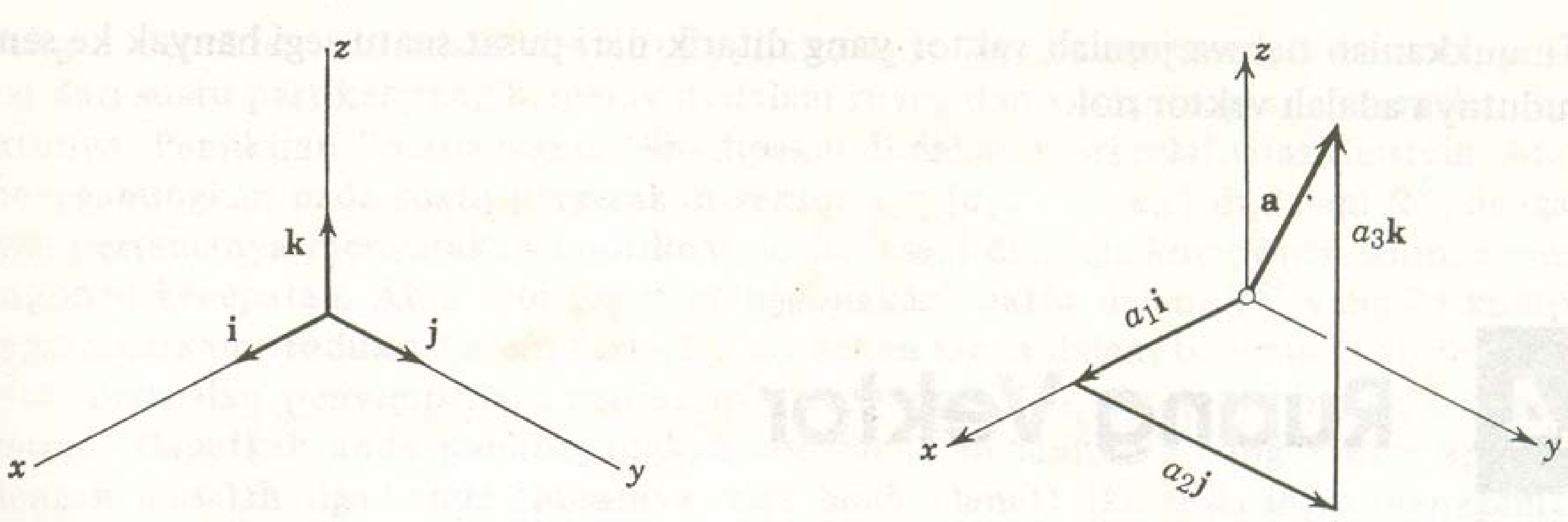
Sifat-sifat

1. $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$
2. $(c + k)\mathbf{a} = c\mathbf{a} + k\mathbf{a}$
3. $c(k\mathbf{a}) = (ck)\mathbf{a}$
4. $1\mathbf{a} = \mathbf{a}$

Vektor-Vektor Satuan : i, j, k

$$\mathbf{a} = [a_1, a_2, a_3] = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$$

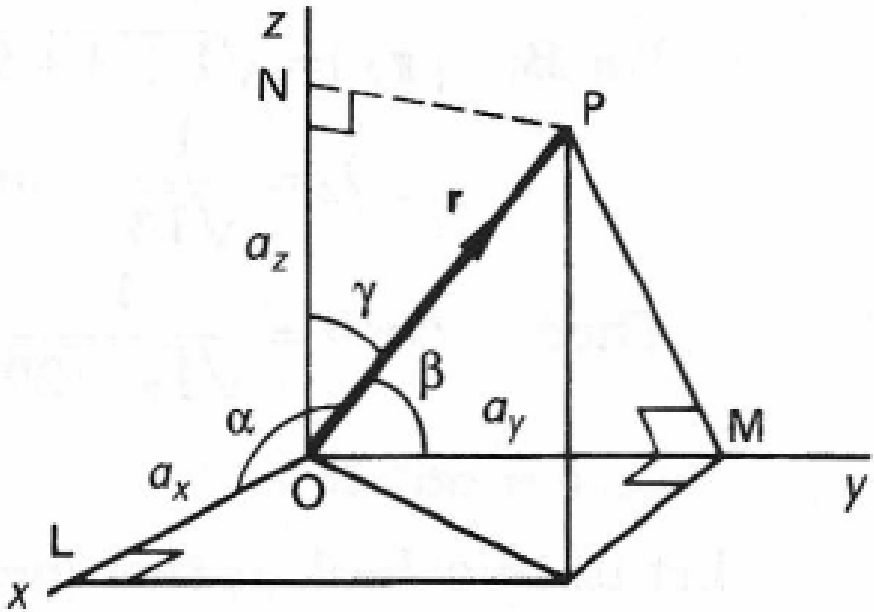
$$\mathbf{i} = [1,0,0], \qquad \mathbf{j} = [0,1,0], \qquad \mathbf{k} = [0,0,1]$$



Cosinus Arah (*Direction Cosines*)

If $\overline{OP} = \mathbf{r} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$ then OP $= |\mathbf{r}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$ where $|\mathbf{r}|$ is the modulus of $\mathbf{r}$.

The direction of OP is denoted by stating the direction cosines of the angles made by OP and the three coordinate axes.



$$l = \cos\alpha = \frac{OL}{OP} = \frac{a_x}{|\mathbf{r}|}$$

$$m = \cos\beta = \frac{OM}{OP} = \frac{a_y}{|\mathbf{r}|}$$

$$n = \cos\gamma = \frac{ON}{OP} = \frac{a_z}{|\mathbf{r}|}$$

$$\therefore\ l,\ m,\ n = \cos\alpha,\ \cos\beta,\ \cos\gamma$$

$$l^2 + m^2 + n^2 = 1$$

Contoh :

So, if P is the point (3, 2, 6), then

$$|\mathbf{r}| = \ldots\ldots\ldots\ldots;$$

$$l = \ldots\ldots\ldots\ldots; \quad m = \ldots\ldots\ldots\ldots; \quad n = \ldots\ldots\ldots\ldots$$

$$(|\mathbf{r}|)^2 = 9 + 4 + 36 = 49 \qquad \therefore \ |\mathbf{r}| = 7$$

$$l = \cos\alpha = \tfrac{3}{7} = 0{\cdot}4286$$

$$m = \cos\beta = \tfrac{2}{7} = 0{\cdot}2857$$

$$n = \cos\gamma = \tfrac{6}{7} = 0{\cdot}8571.$$

Sudut antara dua vektor

If the direction cosines of $\mathbf{A}$ are l_1, m_1, n_1 and those of $\mathbf{B}$ are l_2, m_2, n_2, then the angle between the vectors is given by

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2. \qquad (1)$$

Contoh :

If $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{B} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, we can find the direction cosines of each and hence θ which is …………

For **A**: $|\mathbf{r}_1| = \sqrt{4+9+16} = \sqrt{29}$

$$\therefore\ l_1 = \frac{2}{\sqrt{29}};\quad m_1 = \frac{3}{\sqrt{29}};\quad n_1 = \frac{4}{\sqrt{29}}$$

For **B**: $|\mathbf{r}_2| = \sqrt{1+4+9} = \sqrt{14}$

$$\therefore\ l_2 = \frac{1}{\sqrt{14}};\quad m_2 = \frac{-2}{\sqrt{14}};\quad n_2 = \frac{3}{\sqrt{14}}$$

Then $\cos\theta = \dfrac{1}{\sqrt{14\times 29}}\{2-6+12\} = 0{\cdot}3970$

$$\therefore\ \theta = 66^\circ\ 36'$$

Perkalian skalar antara dua vektor (*dot product*)

If **A** and **B** are two vectors, the scalar product of **A** and **B** is defined as

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta \quad (2)$$

where θ is the angle between the two vectors. If $\mathbf{A} \cdot \mathbf{B} = 0$ then $\mathbf{A} \perp \mathbf{B}$.

If we consider the *scalar products of the unit vectors* **i**, **j**, **k**, which are mutually perpendicular, then

$$\mathbf{i} \cdot \mathbf{j} = (1)(1)\cos 90^\circ = 0 \qquad \therefore\ \mathbf{i} \cdot \mathbf{j} = \mathbf{j} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{i} = 0$$

and $$\mathbf{i} \cdot \mathbf{i} = (1)(1)\cos 0^\circ = 1 \qquad \therefore\ \mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1.$$

In general, if $\mathbf{A} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$ and $\mathbf{B} = b_x\mathbf{i} + b_y\mathbf{j} + b_z\mathbf{k}$ then $\mathbf{A} \cdot \mathbf{B} = a_x b_x + a_y b_y + a_z b_z$ which is, of course, a scalar quantity.

Contoh :

So, if $\mathbf{A} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{B} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, then

$\mathbf{A} \cdot \mathbf{B} = \ldots\ldots\ldots\ldots$

$$\boxed{\mathbf{A} \cdot \mathbf{B} = 2 - 6 + 20 = 16}$$

Also, since $\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$, we can determine the angle θ between the vectors. In this case $\theta = \ldots\ldots\ldots\ldots$

$$\mathbf{A} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k} \qquad \therefore\ A = |\mathbf{A}| = \sqrt{4+9+16} = \sqrt{29}$$

$$\mathbf{B} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} \qquad \therefore\ B = |\mathbf{B}| = \sqrt{1+4+25} = \sqrt{30}$$

We have already found that $\mathbf{A} \cdot \mathbf{B} = 16$ and $\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$

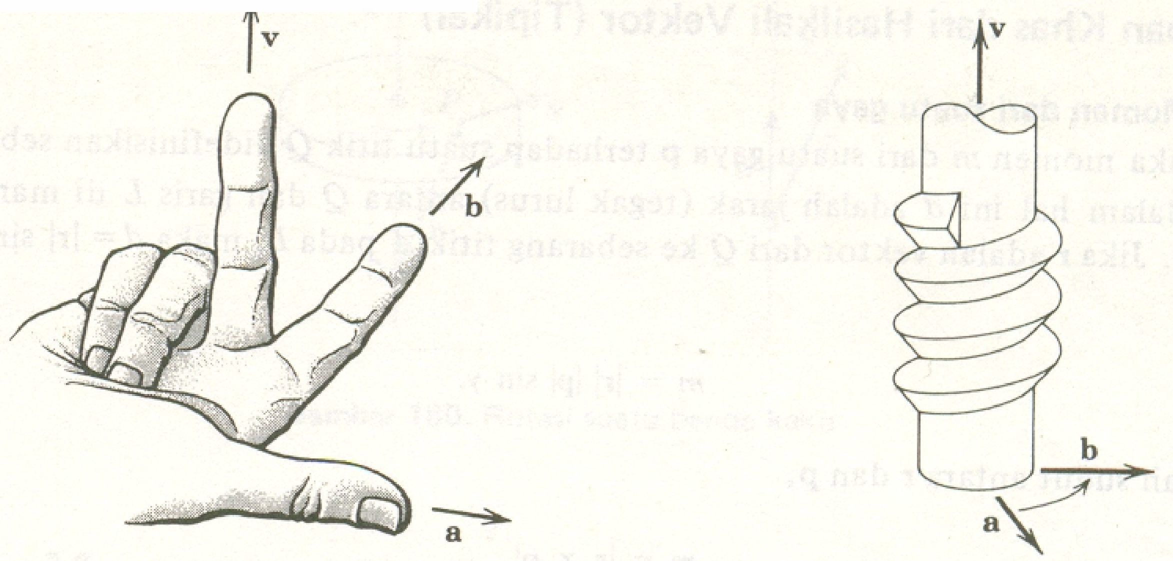
$$\therefore\ 16 = \sqrt{29}\,\sqrt{30}\cos\theta \qquad \therefore\ \cos\theta = 0{\cdot}5425 \qquad \therefore\ \theta = 57^\circ\, 9'$$

Perkalian Vektor (perkalian silang)

If $\mathbf{A} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$ and $\mathbf{B} = b_x\mathbf{i} + b_y\mathbf{j} + b_z\mathbf{k}$ the vector product $\mathbf{A} \times \mathbf{B}$ has magnitude $|\mathbf{A} \times \mathbf{B}| = AB\sin\theta$ in the direction perpendicular to $\mathbf{A}$ and $\mathbf{B}$ such that $\mathbf{A}$, $\mathbf{B}$ and $(\mathbf{A} \times \mathbf{B})$ form a right-handed set.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$|\mathbf{A} \times \mathbf{B}| = AB\sin\theta.$$



If we consider the *vector products of the unit vectors*, **i**, **j**, **k**, then

$$\mathbf{i} \times \mathbf{j} = (1)(1)\sin 90^\circ \mathbf{k} = \mathbf{k}$$

$$\mathbf{j} \times \mathbf{k} = \mathbf{i}, \quad \mathbf{k} \times \mathbf{i} = \mathbf{j}$$

Note that

$$\mathbf{j} \times \mathbf{i} = -(\mathbf{i} \times \mathbf{j}) = -\mathbf{k}$$

$$\mathbf{k} \times \mathbf{j} = -\mathbf{i}, \quad \mathbf{i} \times \mathbf{k} = -\mathbf{j}$$

Also

$$\mathbf{i} \times \mathbf{i} = (1)(1)\sin 0^\circ \mathbf{n} = \mathbf{0}$$

$$\mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$$

Contoh :

Then, if $\mathbf{A} = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$

$$\mathbf{A} \times \mathbf{B} = \ldots\ldots\ldots\ldots$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 4 \\ 2 & -3 & -2 \end{vmatrix}$$

$$= \mathbf{i}(4 + 12) - \mathbf{j}(-6 - 8) + \mathbf{k}(-9 + 4) = 16\mathbf{i} + 14\mathbf{j} - 5\mathbf{k}$$

Contoh :

Therefore, the angle between the vectors **A** and **B**

$$\theta = \ldots\ldots\ldots\ldots\ldots$$

$$\mathbf{A} = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k};\quad \mathbf{B} = 2\mathbf{i} - 3\mathbf{j} - 2\mathbf{k};\quad \text{and}\quad \mathbf{A} \times \mathbf{B} = 16\mathbf{i} + 14\mathbf{j} - 5\mathbf{k}$$

$$\therefore\quad |\,\mathbf{A} \times \mathbf{B}\,| = \sqrt{16^2 + 14^2 + 5^2} = \sqrt{477} = 21{\cdot}84$$

$$A = |\,\mathbf{A}\,| = \sqrt{3^2 + 2^2 + 4^2} = \sqrt{29} = 5{\cdot}385$$

$$B = |\,\mathbf{B}\,| = \sqrt{2^2 + 3^2 + 2^2} = \sqrt{17} = 4{\cdot}123$$

$$\therefore\quad 21{\cdot}84 = (5{\cdot}385)(4{\cdot}123)\sin\theta$$

$$\therefore\quad \sin\theta = 0{\cdot}9838 \qquad \therefore\quad \theta = 79^\circ\, 40'$$

Hasil Kali Tripel Skalar (*Triple Product*)

If **A**, **B**, **C** are three vectors, the scalar formed by the product $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ is called the scalar triple product.

$$\text{If} \quad \mathbf{A} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}; \quad \mathbf{B} = b_x\mathbf{i} + b_y\mathbf{j} + b_z\mathbf{k}; \quad \mathbf{C} = c_x\mathbf{i} + c_y\mathbf{j} + c_z\mathbf{k};$$

$$\text{then} \qquad \mathbf{B} \times \mathbf{C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

$$\therefore \quad \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}) \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

Hasil Kali Tripel Skalar (*Triple Product*)

Multiplying the top row by the external bracket and remembering that

$$\mathbf{i}\cdot\mathbf{j}=\mathbf{j}\cdot\mathbf{k}=\mathbf{k}\cdot\mathbf{i}=0 \quad \text{and} \quad \mathbf{i}\cdot\mathbf{i}=\mathbf{j}\cdot\mathbf{j}=\mathbf{k}\cdot\mathbf{k}=1$$

we have

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})=\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} \tag{6}$$

Contoh :

If $\mathbf{A}=2\mathbf{i}-3\mathbf{j}+4\mathbf{k};\ \mathbf{B}=\mathbf{i}-2\mathbf{j}-3\mathbf{k};\ \mathbf{C}=2\mathbf{i}+\mathbf{j}+2\mathbf{k};$

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})=\begin{vmatrix} 2 & -3 & 4 \\ 1 & -2 & -3 \\ 2 & 1 & 2 \end{vmatrix}$$

$$=2(-4+3)+3(2+6)+4(1+4)=42$$