Fungsi Implisit

- Bila z sebagai fungsi dari x dan y diberikan dalam bentuk implisit :

  F(x,y,z)=0

  Untuk mendapatkan $\dfrac{\partial z}{\partial x}$

Fungsi F diturunkan terhadap x dan menerapkan ***Aturan Rantai***

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x} = -\frac{\partial F / \partial x}{\partial F / \partial z}$$

$$\frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial y} = 0$$

$$\frac{\partial z}{\partial x} = -\frac{\partial F / \partial y}{\partial F / \partial z}$$

Contoh : $$F(x,y,z) = x^2 + 3xy - 2y^2 + 3xz + z^2 = 0$$

Carilah $\dfrac{\partial z}{\partial x}$ dan $\dfrac{\partial z}{\partial y}$

Penyelesaian :

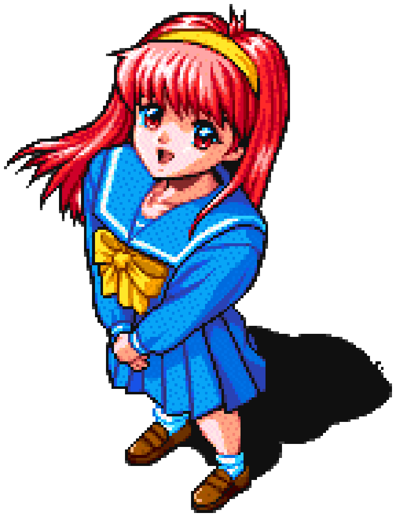
$$(i)\quad \frac{\partial F}{\partial x} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial x} = (2x + 3y + 3z) + (3x + 2z)\frac{\partial z}{\partial x} = 0$$

$$(ii)\quad \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial y} = (3x - 4y) + (3x + 2z)\frac{\partial z}{\partial y} = 0$$

$$\frac{\partial z}{\partial x} = -\frac{\partial F/\partial x}{\partial F/\partial z} = -\frac{2x + 3y + 3z}{3x + 2z} \quad \text{dan} \quad \frac{\partial z}{\partial y} = -\frac{\partial F/\partial y}{\partial F/\partial z} = -\frac{3x - 4y}{3x + 2z}$$

Jacobian

Bila f(x,y,u,v) dan g(x,y,u,v)



Matrik Jacobian $J\left(\frac{f,g}{u,v}\right) = \begin{bmatrix} \frac{\partial f}{\partial u} & \frac{\partial f}{\partial v} \\ \frac{\partial g}{\partial u} & \frac{\partial g}{\partial v} \end{bmatrix}$

Determinan Jacobian $J\left(\frac{f,g}{u,v}\right) = \begin{vmatrix} \frac{\partial f}{\partial u} & \frac{\partial f}{\partial v} \\ \frac{\partial g}{\partial u} & \frac{\partial g}{\partial v} \end{vmatrix}$

Bentuk umum matrik Jacobian

$$\mathbf{J} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix}$$

Contoh :

$$x = r\cos\theta, \quad y = r\sin\theta$$

Tentukan determinan Jacobian.

Penyelesaian :

$$\frac{\partial x}{\partial r} = \cos\theta, \quad \frac{\partial x}{\partial \theta} = -r\sin\theta$$

$$\frac{\partial y}{\partial r} = \sin\theta, \quad \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$\text{Matrik Jacobian} \quad J\left(\frac{x,y}{r,\theta}\right) = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}$$

$$\text{Determinan } \textit{Jacobian} \quad J\left(\frac{x,y}{r,\theta}\right) = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r$$