Titik maksimum atau minimum

Mencari titik maksimum atau minimum relatif dari fungsi *f(x,y)* :

$$\frac{\partial f}{\partial x} = 0 \quad dan \quad \frac{\partial f}{\partial y} = 0$$

$$\Delta = \left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$$

$\Delta < 0$ Titik Pelana (saddle point)

$\Delta = 0$ tidak dapat disimpulkan

$\Delta > 0$ dan $\frac{\partial^2 f}{\partial x^2} > 0$ Titik minimum

$\Delta > 0$ dan $\frac{\partial^2 f}{\partial x^2} < 0$ Titik maksimum

Contoh :

$$f(x,y) = x^2 + y^2 - 4x + 6y + 25$$

Carilah titik maksimum atau minimum

Penyelesaian :

$$\frac{\partial f}{\partial x} = 2x - 4 = 0 \qquad \frac{\partial f}{\partial y} = 2y - 6 = 0$$

$$x = 2, \qquad y = 3$$

$$\frac{\partial^2 f}{\partial x^2} = 2, \qquad \frac{\partial^2 f}{\partial y^2} = 2 \qquad \frac{\partial^2 f}{\partial x \partial y} = 0$$

$$\Delta = \left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = (2)(2) - 0 = 4$$

$$\Delta > 0, \qquad \frac{\partial^2 f}{\partial x^2} > 0$$

Jadi titik (2,3) merupakan titik minimum

Pada titik $(-1,-2)$

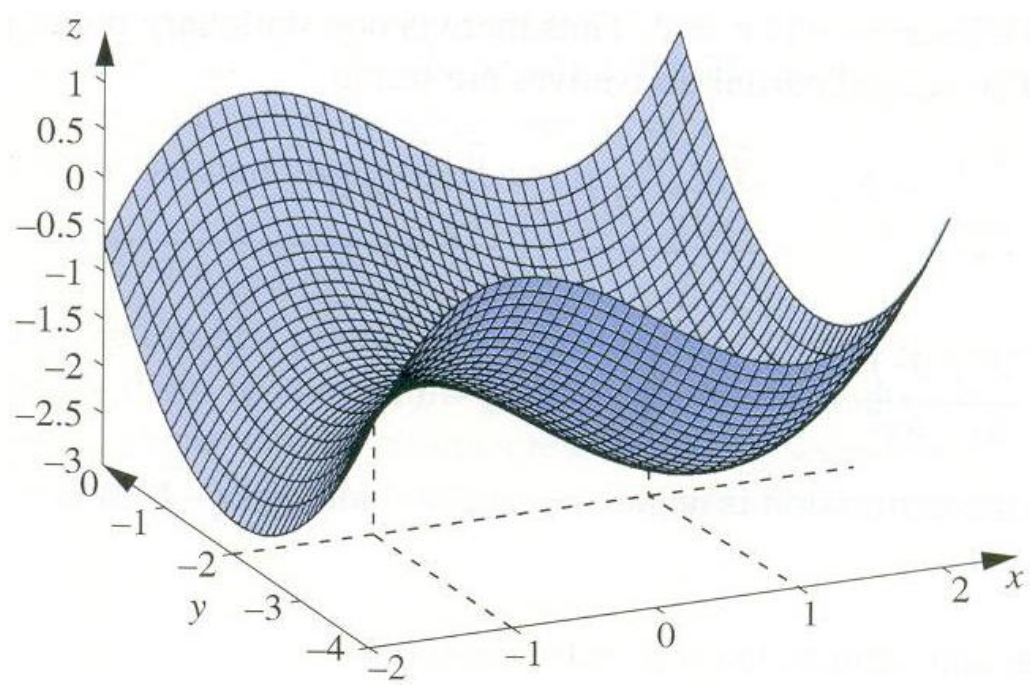
$$\Delta = \left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = (2x)(1) - 0 = 2x = -2, \qquad \Delta < 0,$$

Jadi titik (-1,-2) merupakan titik pelana (sadle point)

Fungsi

$$f(x,y) = \frac{x^3}{3} - x + \frac{y^2}{2} + 2y$$

mempunyai titik minimum di $(1,-2)$ dan titik pelana (sadle point) di $(-1,-2)$



Contoh : $$f(x,y)=\frac{x^3}{3}-x+\frac{y^2}{2}+2y$$

Carilah titik stasioner (maksimum atau minimum)

Penyelesaian :

$$\frac{\partial f}{\partial x}=x^2-1=0 \qquad \frac{\partial f}{\partial y}=y+2=0$$

$$x=1, \qquad y=-2, \text{ dan} \qquad x=-1, \qquad y=-2$$

$$\frac{\partial^2 f}{\partial x^2}=2x, \qquad \frac{\partial^2 f}{\partial y^2}=1 \qquad \frac{\partial^2 f}{\partial x \partial y}=0$$

Pada titik (1,–2)

$$\Delta=\left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right)-\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2=(2x)(1)-0=2x=2$$

$$\Delta>0, \qquad \frac{\partial^2 f}{\partial x^2}=2x=2>0$$

Jadi titik (1,-2) merupakan titik minimum

Metode Lagrange

- Untuk menentukan harga ekstrim dari fungsi :
  - f(x,y)
  - dengan syarat g(x,y,z)=0

$$F(x,y,z) = f(x,y,z) + \lambda g(x,y,z)$$

$$\frac{\partial f}{\partial x} + \lambda \frac{\partial g}{\partial x} = 0, \quad \frac{\partial f}{\partial y} + \lambda \frac{\partial g}{\partial y} = 0, \quad \frac{\partial f}{\partial z} + \lambda \frac{\partial g}{\partial z} = 0$$

$$\lambda = \text{pengali Lagrange}$$

$$f(x,y,z) = x^2 + y^2 + z^2$$

Contoh : Tentukan minimum *f(x,y)* dengan syarat:

$$2x\text{-}y + 2z\text{-}16 = 0$$

Penyelesaian :

$$F(x,y,z) = x^2 + y^2 + z^2 + \lambda(2x - y + 2z - 16)$$

$$\frac{\partial F}{\partial x} = 0, \quad 2x + 2\lambda = 0, \quad x = -\lambda$$

$$\frac{\partial F}{\partial y} = 0, \quad 2y - \lambda = 0, \quad y = \frac{1}{2}\lambda$$

$$\frac{\partial F}{\partial z} = 0, \quad 2z + 2\lambda = 0, \quad z = -\lambda$$

$$\text{Substitusi : } 2(-\lambda) - \frac{1}{2}\lambda + 2(-\lambda) - 16 = 0, \quad \lambda = -\frac{32}{9}$$

$$x = \frac{32}{9}, \quad y = -\frac{16}{9}, \quad z = \frac{32}{9}$$

$$f(x,y,z) \text{ minimum} \quad \left(\frac{32}{9}\right)^2 + \left(-\frac{16}{9}\right)^2 + \left(\frac{32}{9}\right)^2 = \frac{2304}{81}$$