Integral Lipat Dua

$$\iint_R f(x,y)\,dA \quad \Rightarrow R \text{ disebut daerah integrasi}$$

$$\iint_R f(x,y)\,dA = \int_{y=y_1}^{y=y_2}\int_{x=x_1}^{x=x_2} f(x,y)\,dx\,dy \quad \text{(iterated integral)}$$

Untuk menyelesaikannya dimulai dari bagian dalam

$$\int_{y=y_1}^{y=y_2} \left(\int_{x=x_1}^{x=x_2} f(x,y)\,dx \right) dy$$

(1) (2)

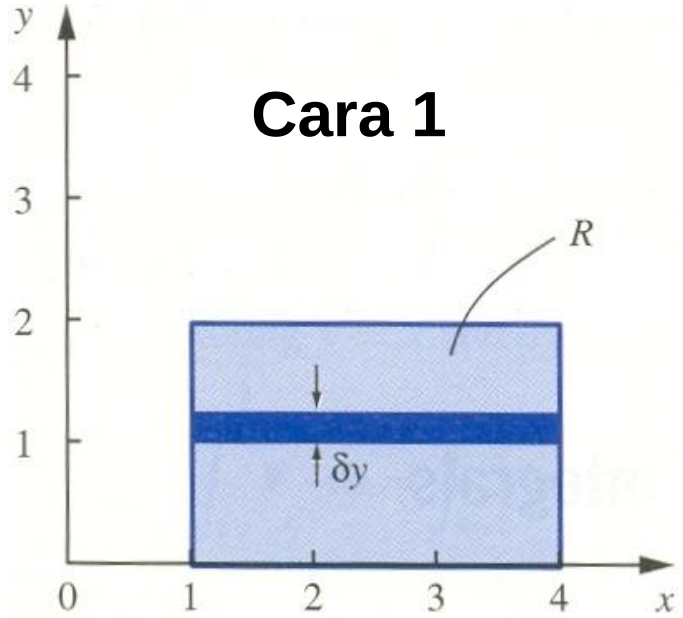
Daerah integrasi harus ditentukan, misalnya berbentuk persegi $x_1 \le x \le x_2$, $y_1 \le y \le y_2$

Contoh:

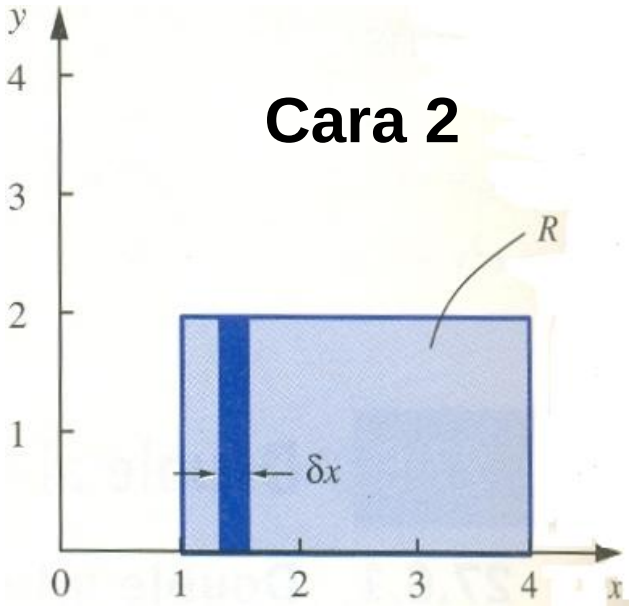
a. Sket daerah R dalam bidang xy yang dibatasi oleh $1 \le x \le 4$ *dan* $0 \le y \le 2$

b. Hitunglah $\int_{y=0}^{y=2} \int_{x=1}^{x=4} x + 2y \, dxdy$

Cara 1



Cara 2



Cara 1

$$\int_{y=0}^{y=2}\int_{x=1}^{x=4} x+2y\,dxdy = \int_{y=0}^{y=2}\left[\frac{x^2}{2}+2yx\right]_{x=1}^{x=4} dy$$

$$= \int_{y=0}^{y=2} 8+8y-\frac{1}{2}-2y\,dy$$

$$= \int_{y=0}^{y=2} \frac{15}{2}+6y dy = \left[\frac{15}{2}y+3y^2\right]_{y=0}^{y=2} = 27$$

Cara 2

$$\int_{y=0}^{y=2}\int_{x=1}^{x=4} x+2y\,dxdy = \int_{x=1}^{x=4}\int_{y=0}^{y=2} x+2y\,dydx$$

$$= \int_{x=1}^{x=4}\left[xy+y^2\right]_{y=0}^{y=2} dx = \int_{x=1}^{x=4} 2x+4\,dx$$

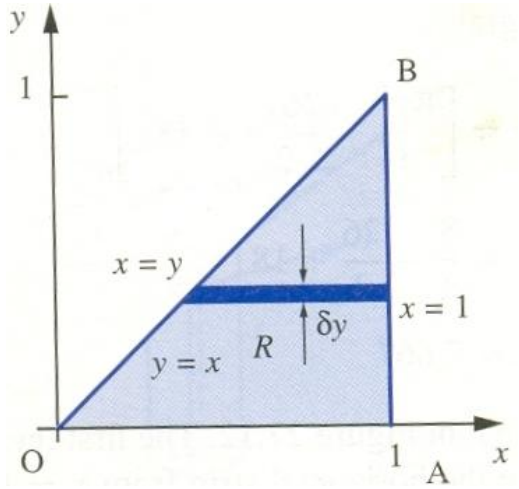
$$= \left[x^2+4x\right]_{x=1}^{x=4} = 27$$

Contoh:

Hitunglah integral rangkap $f(x,y) = x^2 + 3xy$ pada daerah yang didefinisikan pada gambar

Cara 1

$$\int_{y=0}^{y=2}\int_{x=y}^{x=1} x^2 + 3xy\, dxdy = \int_{y=0}^{y=2}\left[\frac{x^3}{3} + \frac{3x^2y}{2}\right]_{x=y}^{x=1} dy$$



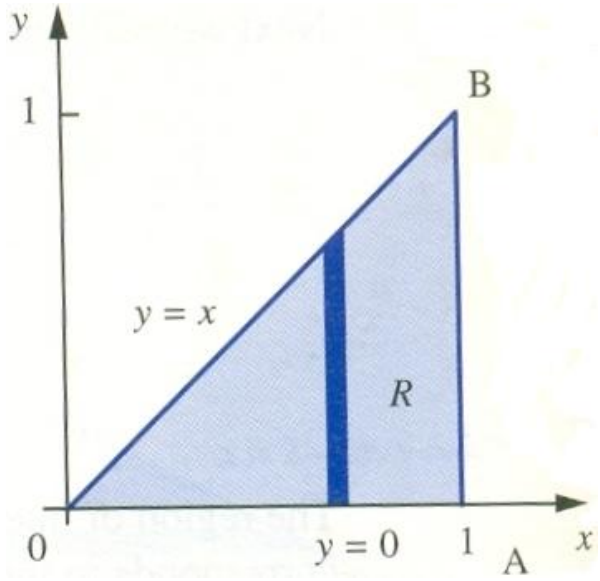
$$= \int_{y=0}^{y=2}\left(\frac{1}{3} + \frac{3y}{2}\right) - \left(\frac{y^3}{3} + \frac{3y^3}{2}\right) dy$$

$$= \int_{y=0}^{y=1} \frac{1}{3} + \frac{3y}{2} - \frac{11y^3}{6}\, dy = \left[\frac{1}{3}y + \frac{3y^2}{4} - \frac{11y^4}{24}\right]_{y=0}^{y=1}$$

$$= \frac{1}{3} + \frac{3}{4} - \frac{11}{24} = \frac{15}{24} = \frac{5}{8}$$

Cara 2

$$\int_{y=0}^{y=2}\int_{x=y}^{x=1} x^2 + 3xy\,dxdy = \int_{x=0}^{x=1}\int_{y=0}^{y=x} x^2 + 3xy\,dydx$$



$$= \int_{x=0}^{x=1}\left[\frac{x^2y}{3} + \frac{3xy^2}{2}\right]_{y=0}^{y=x} dx$$

$$= \int_{x=0}^{x=1} x^3 + \frac{3x^3}{2}\,dx = \int_{x=0}^{x=1} \frac{5x^3}{2}\,dy$$

$$= \left[\frac{5x^4}{8}\right]_{x=0}^{x=1} = \frac{5}{8}$$

Aplikasi Integral Lipat Dua

- Luas daerah datar dengan integral lipat dua

$$\text{Bila } f(x,y)=1, \text{ maka } \iint_R f(x,y)\,dA = \iint_R dA$$

Contoh:

Tentukanlah luas daerah yang dibatasi oleh parabola $y = x^2$ dan garis $y = 2x + 3$

Penyelesaian :

Titik potong dari kedua persamaan tersebut (–1,1) dan (3,9)

$$A = \int_{-1}^{3}\int_{x^2}^{2x+3} dydx = \int_{-1}^{3}\left(2x+3-x^2\right)dx$$

$$= \left[x^2 + 3x - \frac{1}{3}x^3 \right]_{-1}^{3} = \frac{32}{3}$$