

Integral Lipat Tiga

$$\iiint_R f(x, y, z) dV \Rightarrow R \text{ disebut daerah integrasi}$$

$$\iiint_R f(x, y, z) dV = \int_{x=x_1}^{x=x_2} \int_{y=y_1}^{y=y_2} \int_{z=z_1}^{z=z_2} f(x, y, z) dz dy dx$$

Untuk menyelesaikannya dimulai dari bagian dalam

$$\int_{x=x_1}^{x=x_2} \left[\int_{y=y_1}^{y=y_2} \left(\int_{z=z_1}^{z=z_2} f(x, y, z) dz \right) dy \right] dx$$

Contoh :

$$I = \int_{x=0}^{x=1} \int_{y=0}^{y=1} \int_{z=0}^{z=1} x + y + z \, dz \, dy \, dx$$

$$I = \int_{x=0}^{x=1} \left[\int_{y=0}^{y=1} \left(\int_{z=0}^{z=1} x + y + z \, dz \right) dy \right] dx$$

$$= \int_{x=0}^{x=1} \left[\int_{y=0}^{y=1} \left[xz + yz + \frac{1}{2} z^2 \right]_0^1 dy \right] dx$$

$$= \int_{x=0}^{x=1} \left[\int_{y=0}^{y=1} x + y + \frac{1}{2} dy \right] dx = \int_{x=0}^{x=1} \left[xy + \frac{1}{2} y^2 + \frac{1}{2} y \right]_0^1 dx$$

$$= \int_{x=0}^{x=1} x + \frac{1}{2} + \frac{1}{2} dx = \left[\frac{1}{2} x^2 + x \right]_0^1 = \frac{3}{2}$$

Pusat Massa

Dalam koordinat siku - siku (cartesian) massa dari benda R

dengan densitas $\delta(x,y,z)$ diberikan oleh : $m = \iiint_R \delta(x,y,z)dV$

Koordinat pusat massanya $(\bar{x}, \bar{y}, \bar{z})$ adalah :

$$\bar{x} = M_{yz} / m, \quad \bar{y} = M_{xz} / m, \quad \bar{z} = M_{xy} / m$$

$$M_{yz} = \iiint_R \delta(x,y,z)x dV$$

$$M_{xz} = \iiint_R \delta(x,y,z)y dV$$

$$M_{xy} = \iiint_R \delta(x,y,z)z dV$$

Momen Inersia Benda

Momen Inersia dari benda R densitas (x, y, z) berturut - turut terhadap sumbu X , sumbu Y dan sumbu Z didefinisikan :

$$I_x = \iiint_R (y^2 - z^2) \delta(x, y, z) dV$$

$$I_y = \iiint_R (z^2 - x^2) \delta(x, y, z) dV$$

$$I_z = \iiint_R (x^2 - y^2) \delta(x, y, z) dV$$