

Jawaban contoh soal Transformasi Laplace

1. Hitung: $L [10 \sin 4t + 4t^2]$

Jawab:

$$\begin{aligned}L [10 \sin 4t + 4t^2] &= L[10 \sin 4t] + L[4t^2] \\&= 10L[\sin 4t] + 4L[t^2] \\&= 10 \cdot \frac{4}{s^2 + 4^2} + 4 \cdot \frac{\Gamma(3)}{s^3} \\&= \frac{40}{s^2 + 16} + \frac{4 \cdot 2!}{s^3} \\&= \frac{40}{s^2 + 16} + \frac{8}{s^3}\end{aligned}$$

2. Hitung : $L[e^{5t} (\sin 2t - \sin 4t)]$

Jawab :

$$L[e^{5t} (\sin 2t - \sinh 4t)] = L[e^{5t} \sin 2t] - L[e^{5t} \sinh 4t] \dots\dots(1)$$

$$L[\sin 2t] = \frac{2}{s^2 + 4}$$

$$\begin{aligned}L[e^{5t} \sin 2t] &= \frac{2}{(s-5)^2 + 4} \\&= \frac{2}{s^2 - 10s + 25 + 4} \\&= \frac{2}{s^2 - 10s + 29} \dots\dots\dots(2)\end{aligned}$$

$$L[\sinh 4t] = \frac{4}{s^2 - 16}$$

$$\begin{aligned}L[e^{5t} \sinh 4t] &= \frac{4}{(s-5)^2 - 16} \\&= \frac{4}{s^2 - 10s + 25 - 16} \\&= \frac{4}{s^2 - 10s + 9} \dots\dots\dots(3)\end{aligned}$$

Sehingga persamaan (2) dan (3) disubstitusikan pada persamaan (1), sehingga

$$L[e^{5t} (\sin 2t - \sinh 4t)] = \frac{2}{s^2 - 10s + 29} - \frac{4}{s^2 - 10s + 9}$$

3. Hitung : $L[F(t)]$, jika $F(t) = \begin{cases} 5, 0 < t < 3 \\ 0, t > 3 \end{cases}$

Jawab:

$$\begin{aligned} L[F(t)] &= \int_0^{\infty} e^{-st} F(t) dt \\ &= \int_0^3 e^{-st} \cdot 5 \cdot dt + \int_3^{\infty} e^{-st} \cdot 0 \cdot dt \\ &= \int_0^3 e^{-st} \cdot 5 \cdot dt \\ &= 5 \int_0^3 e^{-st} dt \\ &= -\frac{5}{s} e^{-st} \Big|_0^3 \\ &= \frac{5}{s} (1 - e^{-3s}) \end{aligned}$$

4. Hitung : $L[F(t)]$, jika : $F(t) = \begin{cases} \cos(t - 2\pi/3), t > 2\pi/3 \\ 0, t < 2\pi/3 \end{cases}$

$$\begin{aligned} L[F(t)] &= \int_0^{\infty} e^{-st} F(t) dt \\ &= \int_0^{2\pi/3} e^{-st} \cdot 0 \cdot dt + \int_{2\pi/3}^{\infty} e^{-st} \cos(t - 2\pi/3) dt \\ &= \int_{2\pi/3}^{\infty} e^{-st} \cos(t - 2\pi/3) dt \end{aligned}$$

Subs. $u = (t - 2\pi/3)$, $du = dt$

Batas integrasi, $t = \infty \rightarrow u = \infty$

$t = 2\pi/3 \rightarrow u = 0$

$$\begin{aligned} L[F(t)] &= \int_0^{\infty} e^{-s(u+2\pi/3)} \cos u du \\ &= e^{-2\pi/3} \int_0^{\infty} e^{-su} \cos u du \\ &= e^{-2\pi/3} L[\cos u] \\ &= e^{-2\pi/3} \cdot \frac{s}{s^2 + 1} \\ &= \frac{se^{-2\pi/3}}{s^2 + 1} \end{aligned}$$

5. Hitung : $L[(\frac{t}{4})^2]$!

Jawab:

$$\begin{aligned}L[t^2] &= \frac{\Gamma(3)}{s^3} \\ &= \frac{2!}{s^3} \\ &= \frac{2}{s^3}\end{aligned}$$

$$\begin{aligned}L[(\frac{1}{4})^2] &= 4 \cdot \frac{2}{(\frac{s}{1/4})^3} \\ &= \frac{8}{4^3 \cdot s^3} \\ &= \frac{1}{8s^3}\end{aligned}$$