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$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \tag{6.48}$$

In the above equation, j includes bus i. Expressing this equation in polar form, we have

$$I_i = \sum_{j=1}^{n} |Y_{ij}||V_j| \angle \theta_{ij} + \delta_j \tag{6.49}$$

The complex power at bus i is

$$P_i - jQ_i = V_i^* I_i \tag{6.50}$$

Substituting from (6.49) for I_i in (6.50),

$$P_i - jQ_i = |V_i|\angle -\delta_i \sum_{j=1}^{n} |Y_{ij}||V_j|\angle \theta_{ij} + \delta_j \tag{6.51}$$

Separating the real and imaginary parts,

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos\left(\theta_{ij} - \delta_i + \delta_j\right) \tag{6.52}$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin\left(\theta_{ij} - \delta_i + \delta_j\right) \tag{6.53}$$

$$
\left[\begin{array}{c}
\Delta P_2^{(k)} \\
\vdots \\
\Delta P_n^{(k)} \\
\hline
\Delta Q_2^{(k)} \\
\vdots \\
\Delta Q_n^{(k)}
\end{array}\right]
=
\left[\begin{array}{ccc|ccc}
\frac{\partial P_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_2}{\partial \delta_n}^{(k)} & \frac{\partial P_2}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial P_2}{\partial |V_n|}^{(k)} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial P_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_n}{\partial \delta_n}^{(k)} & \frac{\partial P_n}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial P_n}{\partial |V_n|}^{(k)} \\
\hline
\frac{\partial Q_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_2}{\partial \delta_n}^{(k)} & \frac{\partial Q_2}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial Q_2}{\partial |V_n|}^{(k)} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial Q_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_n}{\partial \delta_n}^{(k)} & \frac{\partial Q_n}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial Q_n}{\partial |V_n|}^{(k)}
\end{array}\right]
\left[\begin{array}{c}
\Delta \delta_2^{(k)} \\
\vdots \\
\Delta \delta_n^{(k)} \\
\hline
\Delta |V_2^{(k)}| \\
\vdots \\
\Delta |V_n^{(k)}|
\end{array}\right]
$$

$$
\begin{bmatrix} \boldsymbol{\Delta P} \\ \boldsymbol{\Delta Q} \end{bmatrix} = \begin{bmatrix} \boldsymbol{J_1} & \boldsymbol{J_2} \\ \boldsymbol{J_3} & \boldsymbol{J_4} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Delta \delta} \\ \boldsymbol{\Delta |V|} \end{bmatrix} \tag{6.54}
$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \tag{6.54}$$

For voltage-controlled buses, the voltage magnitudes are known. Therefore, if m buses of the system are voltage-controlled, m equations involving ΔQ and ΔV and the corresponding columns of the Jacobian matrix are eliminated. Accordingly, there are $n-1$ real power constraints and $n-1-m$ reactive power constraints, and the Jacobian matrix is of order $(2n-2-m) \times (2n-2-m)$. J_1 is of the order $(n-1) \times (n-1)$, J_2 is of the order $(n-1) \times (n-1-m)$, J_3 is of the order $(n-1-m) \times (n-1)$, and J_4 is of the order $(n-1-m) \times (n-1-m)$.

The diagonal and the off-diagonal elements of $\boldsymbol{J_1}$ are

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \tag{6.55}$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{6.56}$$

The diagonal and the off-diagonal elements of $\boldsymbol{J_2}$ are

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}| \cos\theta_{ii} + \sum_{j \neq i} |V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \tag{6.57}$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{6.58}$$

The diagonal and the off-diagonal elements of $\boldsymbol{J_3}$ are

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j \neq i} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \tag{6.59}$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{6.60}$$

The diagonal and the off-diagonal elements of $\boldsymbol{J_4}$ are

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}| \sin \theta_{ii} - \sum_{j \neq i} |V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \tag{6.61}$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{6.62}$$

The terms $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are the difference between the scheduled and calculated values, known as the *power residuals*, given by

$$\Delta P_i^{(k)} = P_i^{sch} - P_i^{(k)} \tag{6.63}$$

$$\Delta Q_i^{(k)} = Q_i^{sch} - Q_i^{(k)} \tag{6.64}$$

The new estimates for bus voltages are

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)} \tag{6.65}$$

$$|V_i^{(k+1)}| = |V_i^{(k)}| + \Delta|V_i^{(k)}| \tag{6.66}$$

The procedure for power flow solution by the Newton-Raphson method is as follows:

1. For load buses, where P_i^{sch} and Q_i^{sch} are specified, voltage magnitudes and phase angles are set equal to the slack bus values, or 1.0 and 0.0, i.e., $|V_i^{(0)}| = 1.0$ and $\delta_i^{(0)} = 0.0$. For voltage-regulated buses, where $|V_i|$ and P_i^{sch} are specified, phase angles are set equal to the slack bus angle, or 0, i.e., $\delta_i^{(0)} = 0$.

2. For load buses, $P_i^{(k)}$ and $Q_i^{(k)}$ are calculated from (6.52) and (6.53) and $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are calculated from (6.63) and (6.64).

3. For voltage-controlled buses, $P_i^{(k)}$ and $\Delta P_i^{(k)}$ are calculated from (6.52) and (6.63), respectively.

4. The elements of the Jacobian matrix (J_1, J_2, J_3, and J_4) are calculated from (6.55) – (6.62).

5. The linear simultaneous equation (6.54) is solved directly by optimally ordered triangular factorization and Gaussian elimination.

6. The new voltage magnitudes and phase angles are computed from (6.65) and (6.66).

7. The process is continued until the residuals $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are less than the specified accuracy, i.e.,

$$\begin{aligned} |\Delta P_i^{(k)}| &\leq \epsilon \\ |\Delta Q_i^{(k)}| &\leq \epsilon \end{aligned} \tag{6.67}$$

Example 6.8

Figure 6.12 shows the one-line diagram of a simple three-bus power system with generators at buses 1 and 3. The magnitude of voltage at bus 1 is adjusted to 1.05 pu. Voltage magnitude at bus 3 is fixed at 1.04 pu with a real power generation of 200 MW. A load consisting of 400 MW and 250 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100 MVA base, and the line charging susceptances are neglected. Obtain the power flow solution by the Gauss-Seidel method including line flows and line losses.

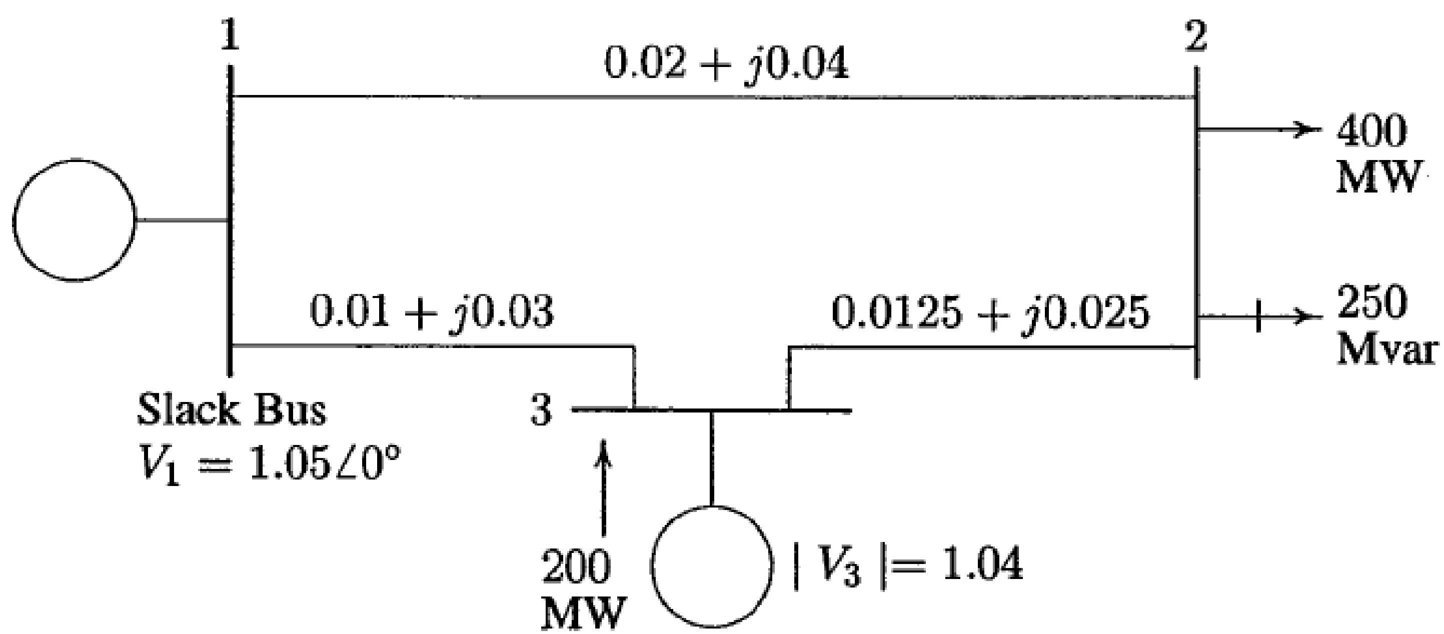

FIGURE 6.12
One-line diagram of Example 6.8 (impedances in pu on 100-MVA base).

Line impedances converted to admittances are $y_{12} = 10 - j20$, $y_{13} = 10 - j30$, and $y_{23} = 16 - j32$. This results in the bus admittance matrix

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

Converting the bus admittance matrix to polar form with angles in radian yields

$$Y_{bus} = \begin{bmatrix} 53.85165\angle -1.9029 & 22.36068\angle 2.0344 & 31.62278\angle 1.8925 \\ 22.36068\angle 2.0344 & 58.13777\angle -1.1071 & 35.77709\angle 2.0344 \\ 31.62278\angle 1.8925 & 35.77709\angle 2.0344 & 67.23095\angle -1.1737 \end{bmatrix}$$

From (6.52) and (6.53), the expressions for real power at bus 2 and 3 and the reactive power at bus 2 are

$$\begin{aligned} P_2 &= |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2^2||Y_{22}|\cos\theta_{22} + \\ &\quad |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3) \\ P_3 &= |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32} - \\ &\quad \delta_3 + \delta_2) + |V_3^2||Y_{33}|\cos\theta_{33} \\ Q_2 &= -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2^2||Y_{22}|\sin\theta_{22} - \\ &\quad |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3) \end{aligned}$$

Elements of the Jacobian matrix are obtained by taking partial derivatives of the above equations with respect to δ_2, δ_3 and $|V_2|$.

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + 2|V_2||Y_{22}|\cos\theta_{22} + |V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}|$$
$$\cos(\theta_{23} - \delta_2 + \delta_3)$$
$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$
$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - 2|V_2||Y_{22}|\sin\theta_{22} -$$
$$|V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \quad \text{pu}$$
$$P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

The slack bus voltage is $V_1 = 1.05\angle 0$ pu, and the bus 3 voltage magnitude is $|V_3| = 1.04$ pu. Starting with an initial estimate of $|V_2^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -4.0 - (-1.14) = -2.8600$$
$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = 2.0 - (0.5616) = 1.4384$$
$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -2.5 - (-2.28) = -0.2200$$

Evaluating the elements of the Jacobian matrix with the initial estimate, the set of linear equations in the first iteration becomes

$$\begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} 54.28000 & -33.28000 & 24.86000 \\ -33.28000 & 66.04000 & -16.64000 \\ -27.14000 & 16.64000 & 49.72000 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, the new bus voltages in the first iteration are

$$
\begin{aligned}
\Delta\delta_2^{(0)} &= -0.045263 & \delta_2^{(1)} &= 0 + (-0.045263) = -0.045263 \\
\Delta\delta_3^{(0)} &= -0.007718 & \delta_3^{(1)} &= 0 + (-0.007718) = -0.007718 \\
\Delta|V_2^{(0)}| &= -0.026548 & |V_2^{(1)}| &= 1 + (-0.026548) = 0.97345
\end{aligned}
$$

Voltage phase angles are in radians. For the second iteration, we have

$$
\begin{bmatrix} -0.099218 \\ 0.021715 \\ -0.050914 \end{bmatrix} = \begin{bmatrix} 51.724675 & -31.765618 & 21.302567 \\ -32.981642 & 65.656383 & -15.379086 \\ -28.538577 & 17.402838 & 48.103589 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \\ \Delta|V_2^{(1)}| \end{bmatrix}
$$

and

$$\begin{aligned}
\Delta\delta_2^{(1)} &= -0.001795 & \delta_2^{(2)} &= -0.045263 + (-0.001795) = -0.04706 \\
\Delta\delta_3^{(1)} &= -0.000985 & \delta_3^{(2)} &= -0.007718 + (-0.000985) = -0.00870 \\
\Delta|V_2^{(1)}| &= -0.001767 & |V_2^{(2)}| &= 0.973451 + (-0.001767) = 0.971684
\end{aligned}$$

For the third iteration, we have

$$\begin{bmatrix} -0.000216 \\ 0.000038 \\ -0.000143 \end{bmatrix} = \begin{bmatrix} 51.596701 & -31.693866 & 21.147447 \\ -32.933865 & 65.597585 & -15.351628 \\ -28.548205 & 17.396932 & 47.954870 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(2)} \\ \Delta\delta_3^{(2)} \\ \Delta|V_2^{(2)}| \end{bmatrix}$$

and

$$\begin{aligned}
\Delta\delta_2^{(2)} &= -0.000038 & \delta_2^{(3)} &= -0.047058 + (-0.0000038) = -0.04706 \\
\Delta\delta_3^{(2)} &= -0.0000024 & \delta_3^{(3)} &= -0.008703 + (-0.0000024) = 0.008705 \\
\Delta|V_2^{(2)}| &= -0.0000044 & |V_2^{(3)}| &= 0.971684 + (-0.0000044) = 0.97168
\end{aligned}$$

The solution converges in 3 iterations with a maximum power mismatch of 2.5×10^{-4} with $V_2 = 0.97168\angle{-2.696°}$ and $V_3 = 1.04\angle{-0.4988°}$. From (6.52) and (6.53), the expressions for reactive power at bus 3 and the slack bus real and reactive powers are

$$
\begin{aligned}
Q_3 &= -|V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) - |V_3||V_2||Y_{32}| \\
&\quad \sin(\theta_{32} - \delta_3 + \delta_2) - |V_3|^2|Y_{33}|\sin\theta_{33} \\
P_1 &= |V_1|^2|Y_{11}|\cos\theta_{11} + |V_1||V_2||Y_{12}|\cos(\theta_{12} - \delta_1 + \delta_2) + |V_1||V_3| \\
&\quad |Y_{13}|\cos(\theta_{13} - \delta_1 + \delta_3) \\
Q_1 &= -|V_1|^2|Y_{11}|\sin\theta_{11} - |V_1||V_2||Y_{12}|\sin(\theta_{12} - \delta_1 + \delta_2) - |V_1||V_3| \\
&\quad |Y_{13}|\sin(\theta_{13} - \delta_1 + \delta_3)
\end{aligned}
$$

Upon substitution, we have

$$
\begin{aligned}
Q_3 &= 1.4617 \quad \text{pu} \\
P_1 &= 2.1842 \quad \text{pu} \\
Q_1 &= 1.4085 \quad \text{pu}
\end{aligned}
$$

Finally, the line flows are calculated in the same manner as the line flow calculations in the Gauss-Seidel method described in Example 6.7, and the power flow diagram is as shown in Figure 6.13.

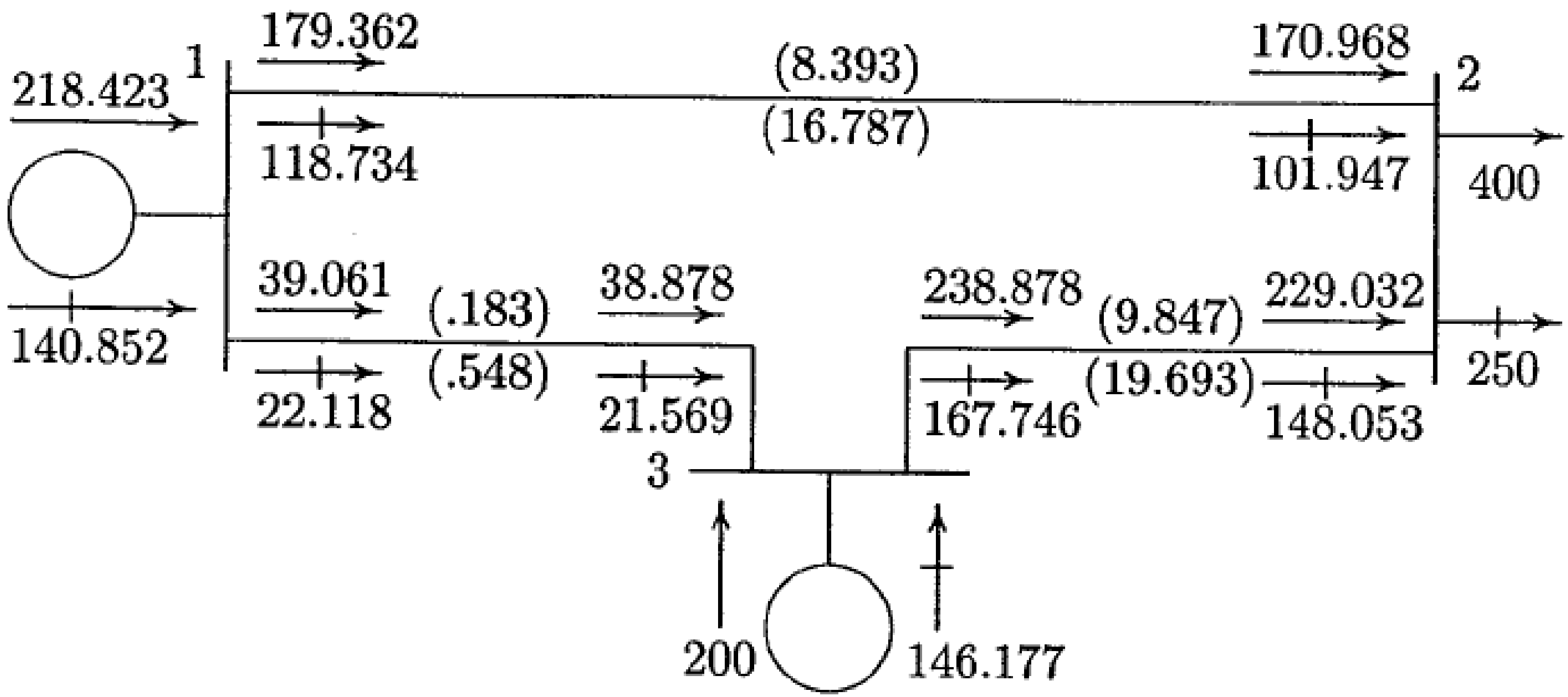

FIGURE 6.13
Power flow diagram of Example 6.8 (powers in MW and Mvar).

6.11. In the two-bus system shown in Figure 59, bus 1 is a slack bus with $V_1 = 1.0\angle 0°$ pu. A load of 100 MW and 50 Mvar is taken from bus 2. The line impedance is $z_{12} = 0.12 + j0.16$ pu on a base of 100 MVA. Using Newton-Raphson method, obtain the voltage magnitude and phase angle of bus 2. Start with an initial estimate of $|V_2|^{(0)} = 1.0$ pu and $\delta_2{}^{(0)} = 0°$. Perform two iterations.

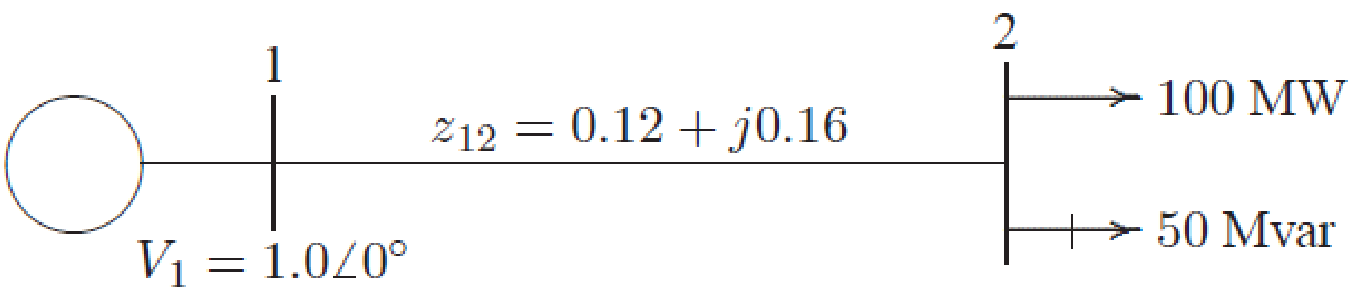

FIGURE 59
One-line diagram for Problem 6.11.

The power flow equation with voltages and admittances expressed in polar form is

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos\left(\theta_{ij} - \delta_i + \delta_j\right)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin\left(\theta_{ij} - \delta_i + \delta_j\right)$$

The bus admittance matrix is

$$Y_{bus} = \begin{bmatrix} 5\angle{-53.13^\circ} & 5\angle 126.87^\circ \\ 5\angle 126.87^\circ & 5\angle{-53.13^\circ} \end{bmatrix}$$

Substituting for admittances, the expression for real and reactive power at bus 2 becomes

$$\begin{aligned} P_2 &= \ \ 5|V_2||V_1|\cos(126.87^\circ - \delta_2 + \delta_1) + 5|V_2|^2\cos(-53.13^\circ) \\ Q_2 &= -5|V_2||V_1|\sin(126.87^\circ - \delta_2 + \delta_1) - 5|V_2|^2\sin(-53.13^\circ) \end{aligned}$$

Partial derivatives of P_2, and Q_2 with respect to $|V_2|$, and δ_2 are

$$\begin{aligned} \frac{\partial P_2}{\partial \delta_2} &= 5|V_2||V_1|\sin(126.87^\circ - \delta_2 + \delta_1) \\ \frac{\partial P_2}{\partial |V_2|} &= 5|V_1|\cos(126.87^\circ - \delta_2 + \delta_1) + 10|V_2|\cos(-53.13^\circ) \\ \frac{\partial Q_2}{\partial \delta_2} &= 5|V_2||V_1|\cos(126.87^\circ - \delta_2 + \delta_1) \\ \frac{\partial Q_2}{\partial |V_2|} &= -5|V_1|\sin(126.87^\circ - \delta_2 + \delta_1) - 10|V_2|\sin(-73.74^\circ) \end{aligned}$$

The load expressed in per units is

$$S_2^{sch} \ = \ -\frac{(100 + j50)}{100} = -1.0 - j0.5 \ \text{ pu}$$

The slack bus voltage is $V_1 = 1.0\angle 0$ pu. Starting with an initial estimate of $|V_2^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\begin{aligned}
\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} &= -1.0 - [5\cos(126.87^\circ) + 5\cos(-53.13^\circ)] \\
&= -1.0 \ \text{pu} \\
\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} &= -0.5 - [-5\sin(126.87^\circ) - 5\sin(-53.13^\circ)] \\
&= -0.5 \ \text{pu}
\end{aligned}$$

The elements of the Jacobian matrix at the initial estimate are

$$\begin{aligned}
J_1^{(0)} &= 5(1)(1)\sin(126.87^\circ) = 4 \\
J_2^{(0)} &= 5(1)\cos(126.87^\circ) + 10(1)\cos(-53.13^\circ) = 3 \\
J_3^{(0)} &= 5(1)(1)\cos(126.87^\circ) = -3 \\
J_4^{(0)} &= -5(1)\sin(126.87^\circ) - 10(1)\sin(-53.13^\circ) = 4
\end{aligned}$$

The set of linear equations in the first iteration becomes

$$\begin{bmatrix} -1.0 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus 2 in the first iteration is

$$\Delta\delta_2^{(0)} = -0.10 \qquad \delta_2^{(1)} = 0 + (-0.10) = -0.10 \text{ radian}$$

$$\Delta|V_2^{(0)}| = -0.2 \qquad |V_2^{(1)}| = 1 + (-0.2) = 0.8 \text{ pu}$$

For the second iteration, we have

$$\Delta P_2^{(1)} = P_2^{sch} - P_2^{(1)} = -1.0 - (-0.7875) = -0.2125 \text{ pu}$$

$$\Delta Q_2^{(1)} = Q_2^{sch} - Q_2^{(1)} = -0.5 - (-0.3844) = -0.1156 \text{ pu}$$

Also, computing the elements of the Jacobian matrix, the set of linear equations in the second iteration becomes

$$\begin{bmatrix} -0.2125 \\ -0.1156 \end{bmatrix} = \begin{bmatrix} 2.9444 & 1.4157 \\ -2.7075 & 2.7195 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta|V_2^{(1)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, voltage at bus 2 in the second iteration is

$$\Delta\delta_2^{(1)} = -0.0350 \qquad \delta_2^{(2)} = -0.1 + (-0.0350) = -0.135 \ \text{radian}$$

$$\Delta|V_2^{(1)}| = -0.0773 \qquad |V_2^{(2)}| = 0.8 + (-0.0773) = 0.7227 \ \text{pu}$$