FAST DECOUPLED POWER FLOW SOLUTION

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$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix}$$

Power system transmission lines have a very high X/R ratio. For such a system, real power changes ΔP are less sensitive to changes in the voltage magnitude and are most sensitive to changes in phase angle $\Delta\delta$. Similarly, reactive power is less sensitive to changes in angle and are mainly dependent on changes in voltage magnitude. Therefore, it is reasonable to set elements J_2 and J_3 of the Jacobian matrix to zero. Thus, (6.54) becomes

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \tag{6.68}$$

or

$$\Delta P = J_1 \Delta\delta = [\frac{\partial P}{\partial \delta}]\, \Delta\delta \tag{6.69}$$

$$\Delta Q = J_4 \Delta|V| = [\frac{\partial Q}{\partial |V|}]\, \Delta|V| \tag{6.70}$$

(6.69) and (6.70) show that the matrix equation is separated into two decoupled equations requiring considerably less time to solve compared to the time required for the solution of (6.54). Furthermore, considerable simplification can be made to eliminate the need for recomputing J_1 and J_4 during each iteration. This procedure results in the decoupled power flow equations developed by Stott and Alsac[75–76]. The diagonal elements of J_1 described by (6.55) may be written as

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) - |V_i|^2|Y_{ii}|\sin\theta_{ii}$$

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) - |V_i|^2 |Y_{ii}| \sin \theta_{ii}$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \tag{6.53}$$

Replacing the first term of the above equation with $-Q_i$, as given by (6.53), results in

$$\begin{aligned} \frac{\partial P_i}{\partial \delta_i} &= -Q_i - |V_i|^2 |Y_{ii}| \sin \theta_{ii} \\ &= -Q_i - |V_i|^2 B_{ii} \end{aligned}$$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Where $B_{ii} = |Y_{ii}| \sin \theta_{ii}$ is the imaginary part of the diagonal elements of the bus admittance matrix. B_{ii} is the sum of susceptances of all the elements incident to bus i. In a typical power system, the self-susceptance $B_{ii} \gg Q_i$, and we may neglect Q_i. Further simplification is obtained by assuming $|V_i|^2 \approx |V_i|$, which yields

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i| B_{ii} \tag{6.71}$$

Under normal operating conditions, $\delta_j - \delta_i$ is quite small. Thus, in (6.56) assuming $\theta_{ii} - \delta_i + \delta_j \approx \theta_{ii}$, the off-diagonal elements of $\boldsymbol{J_1}$ becomes

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j|B_{ij}$$

Further simplification is obtained by assuming $|V_j| \approx 1$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i|B_{ij} \tag{6.72}$$

Similarly, the diagonal elements of $\boldsymbol{J_4}$ described by (6.61) may be written as

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i||Y_{ii}|\sin\theta_{ii} - \sum_{j=1}^{n}|V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

replacing the second term of the above equation with $-Q_i$, as given by (6.53), results in

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i||Y_{ii}|\sin\theta_{ii} + Q_i$$

Again, since $B_{ii} = Y_{ii}\sin\theta_{ii} \gg Q_i$, Q_i may be neglected and (6.61) reduces to

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i|B_{ii} \tag{6.73}$$

Likewise in (6.62), assuming $\theta_{ij} - \delta_i + \delta_j \approx \theta_{ij}$ yields

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i| B_{ij} \tag{6.74}$$

With these assumptions, equations (6.69) and (6.70) take the following form

$$\frac{\Delta \boldsymbol{P}}{|V_i|} = -\boldsymbol{B}' \, \Delta\boldsymbol{\delta} \tag{6.75}$$

$$\frac{\Delta \boldsymbol{Q}}{|V_i|} = -\boldsymbol{B}'' \, \Delta|\boldsymbol{V}| \tag{6.76}$$

Here, $\boldsymbol{B}'$ and $\boldsymbol{B}''$ are the imaginary part of the bus admittance matrix $\boldsymbol{Y_{bus}}$. Since the elements of this matrix are constant, they need to be triangularized and inverted only once at the beginning of the iteration. $\boldsymbol{B}'$ is of order of $(n-1)$. For voltage-controlled buses where $|V_i|$ and P_i are specified and Q_i is not specified, the corresponding row and column of $\boldsymbol{Y_{bus}}$ are eliminated. Thus, $\boldsymbol{B}''$ is of order of $(n-1-m)$, where m is the number of voltage-regulated buses. Therefore, in the fast decoupled power flow algorithm, the successive voltage magnitude and phase angle changes are

$$\Delta\boldsymbol{\delta} = -[\boldsymbol{B}']^{-1} \frac{\Delta \boldsymbol{P}}{|\boldsymbol{V}|} \tag{6.77}$$

$$\Delta|\boldsymbol{V}| = -[\boldsymbol{B}'']^{-1} \frac{\Delta \boldsymbol{Q}}{|\boldsymbol{V}|} \tag{6.78}$$

Example 6.8

Figure 6.12 shows the one-line diagram of a simple three-bus power system with generators at buses 1 and 3. The magnitude of voltage at bus 1 is adjusted to 1.05 pu. Voltage magnitude at bus 3 is fixed at 1.04 pu with a real power generation of 200 MW. A load consisting of 400 MW and 250 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100 MVA base, and the line charging susceptances are neglected. Obtain the power flow solution by the Gauss-Seidel method including line flows and line losses.

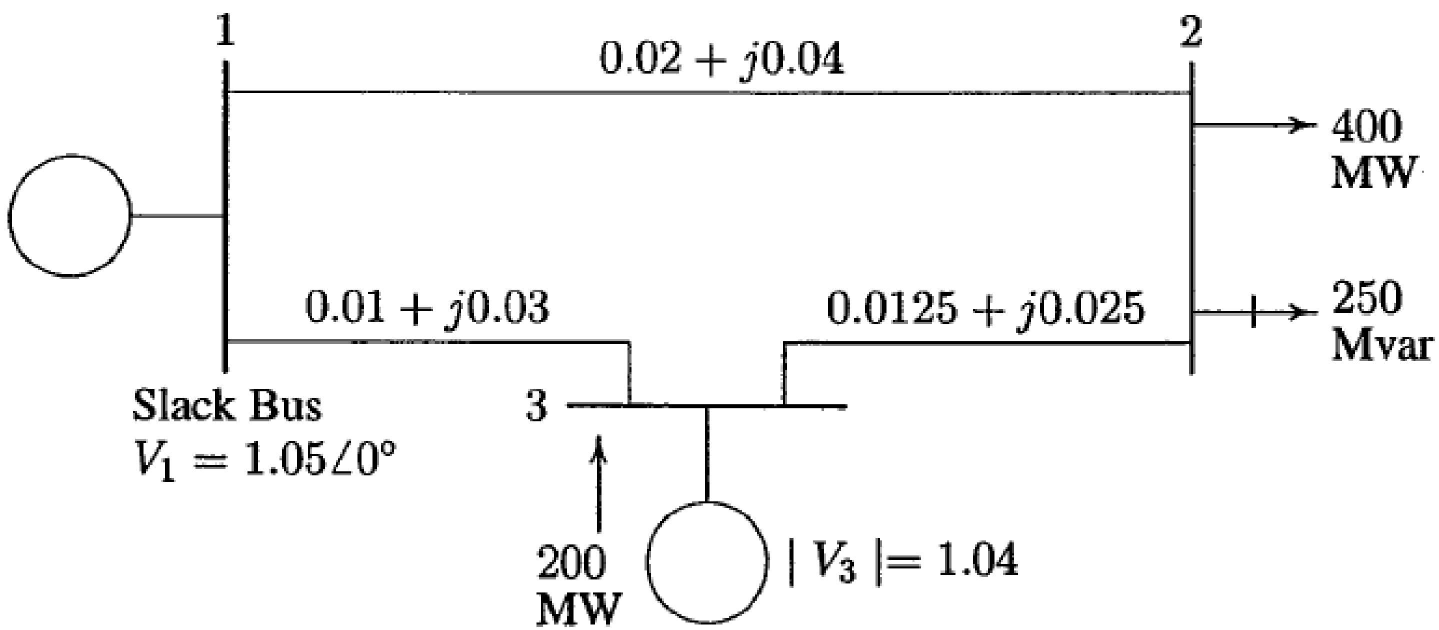

FIGURE 6.12
One-line diagram of Example 6.8 (impedances in pu on 100-MVA base).

Obtain the power flow solution by the fast decoupled method for the system of Example 6.8.

The bus admittance matrix of the system as obtained in Example 6.10 is

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

In this system, bus 1 is the slack bus and the corresponding bus susceptance matrix for evaluation of phase angles $\Delta\delta_2$ and $\Delta\delta_3$ is

$$B' = \begin{bmatrix} -52 & 32 \\ 32 & -62 \end{bmatrix}$$

The inverse of the above matrix is

$$[B']^{-1} = \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix}$$

From (6.52) and (6.53), the expressions for real power at bus 2 and 3 and the reactive power at bus 2 are

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2^2||Y_{22}|\cos\theta_{22}$$
$$+|V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$
$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32}$$
$$-\delta_3 + \delta_2) + |V_3^2||Y_{33}|\cos\theta_{33}$$
$$Q_2 = -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2^2||Y_{22}|\sin\theta_{22}$$
$$-|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \quad \text{pu}$$
$$P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

The slack bus voltage is $V_1 = 1.05\angle 0$ pu, and the bus 3 voltage magnitude is $|V_3| = 1.04$ pu. Starting with an initial estimate of $|V_2^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -4.0 - (-1.14) = -2.86$$

$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = 2.0 - (0.5616) = 1.4384$$

$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -2.5 - (-2.28) = -0.22$$

The fast decoupled power flow algorithm given by (6.77) becomes

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix} = -\begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix} \begin{bmatrix} \frac{-2.8600}{1.0} \\ \frac{1.4384}{1.04} \end{bmatrix} = \begin{bmatrix} -0.060483 \\ -0.008909 \end{bmatrix}$$

Since bus 3 is a regulated bus, the corresponding row and column of $\boldsymbol{B'}$ are eliminated and we get

$$B'' = [-52]$$

From (6.78), we have

$$\Delta|V_2| = -\left[\frac{-1}{52}\right]\left[\frac{-.22}{1.0}\right] = -0.0042308$$

The new bus voltages in the first iteration are

$$\Delta\delta_2^{(0)} = -0.060483 \qquad \delta_2^{(1)} = 0 + (-0.060483) = -0.060483$$
$$\Delta\delta_3^{(0)} = -0.008989 \qquad \delta_3^{(1)} = 0 + (-0.008989) = -0.008989$$
$$\Delta|V_2^{(0)}| = -0.0042308 \qquad |V_2^{(1)}| = 1 + (-0.0042308) = 0.995769$$

The voltage phase angles are in radians. The process is continued until power residuals are within a specified accuracy. The result is tabulated in the table below.

Iter	δ_2	δ_3	$\|V_2\|$	ΔP_2	ΔP_3	ΔQ_2
1	-0.060482	-0.008909	0.995769	-2.860000	1.438400	-0.220000
2	-0.056496	-0.007952	0.965274	0.175895	-0.070951	-1.579042
3	-0.044194	-0.008690	0.965711	0.640309	-0.457039	0.021948
4	-0.044802	-0.008986	0.972985	-0.021395	0.001195	0.365249
5	-0.047665	-0.008713	0.973116	-0.153368	0.112899	0.006657
6	-0.047614	-0.008645	0.971414	0.000520	0.002610	-0.086136
7	-0.046936	-0.008702	0.971333	0.035980	-0.026190	-0.004067
8	-0.046928	-0.008720	0.971732	0.000948	-0.001411	0.020119
9	-0.047087	-0.008707	0.971762	-0.008442	0.006133	0.001558
10	-0.047094	-0.008702	0.971669	-0.000470	0.000510	-0.004688
11	-0.047057	-0.008705	0.971660	0.001971	-0.001427	-0.000500
12	-0.047054	-0.008706	0.971681	0.000170	-0.000163	0.001087
13	-0.047063	-0.008706	0.971684	-0.000458	0.000330	0.000151
14	-0.047064	-0.008706	0.971680	-0.000053	0.000048	-0.000250

Converting phase angles to degrees the final solution is $V_2 = 0.97168\angle -2.696°$ and $V_3 = 1.04\angle -0.4988°$. Using (6.52) and (6.53) as in Example 6.10, the reactive power at bus 3 and the slack bus real and reactive powers are

$$Q_3 = 1.4617 \ \text{pu}$$
$$P_1 = 2.1842 \ \text{pu}$$
$$Q_1 = 1.4085 \ \text{pu}$$

The fast decoupled power flow for this example has taken 14 iterations with the maximum power mismatch of 2.5×10^{-4} pu compared to the Newton-Raphson method which took only three iterations. The highest X/R ratio of the transmission lines in this example is 3. For systems with a higher X/R ratio, the fast decoupled power flow method converges in relatively fewer iterations. However, the number of iterations is a function of system size.

Finally, the line flows are calculated in the same manner as the line flow calculations in the Gauss-Seidel method described in Example 6.7, and the power flow diagram is as shown in Figure 6.13.

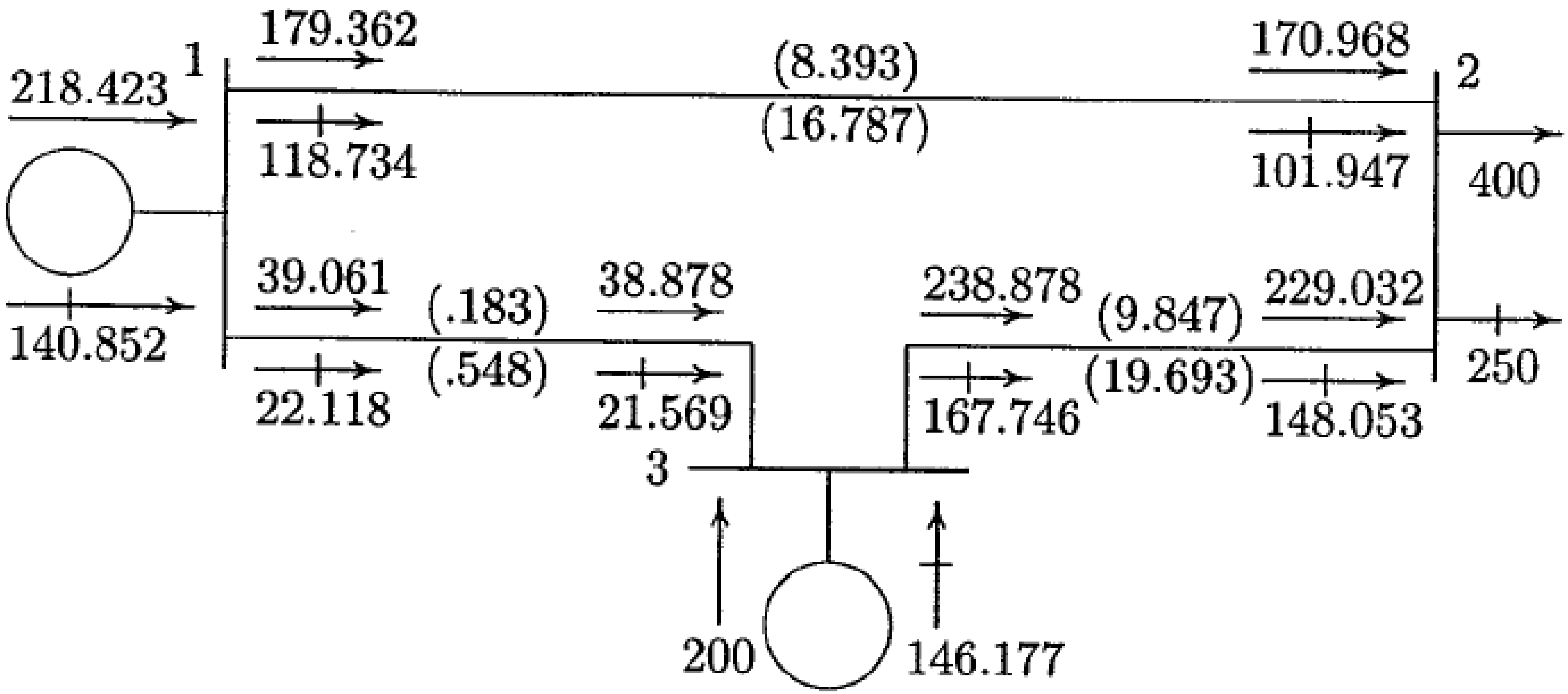

FIGURE 6.13
Power flow diagram of Example 6.8 (powers in MW and Mvar).