THREE-PHASE SHORT CIRCUIT—UNLOADED SYNCHRONOUS MACHINE

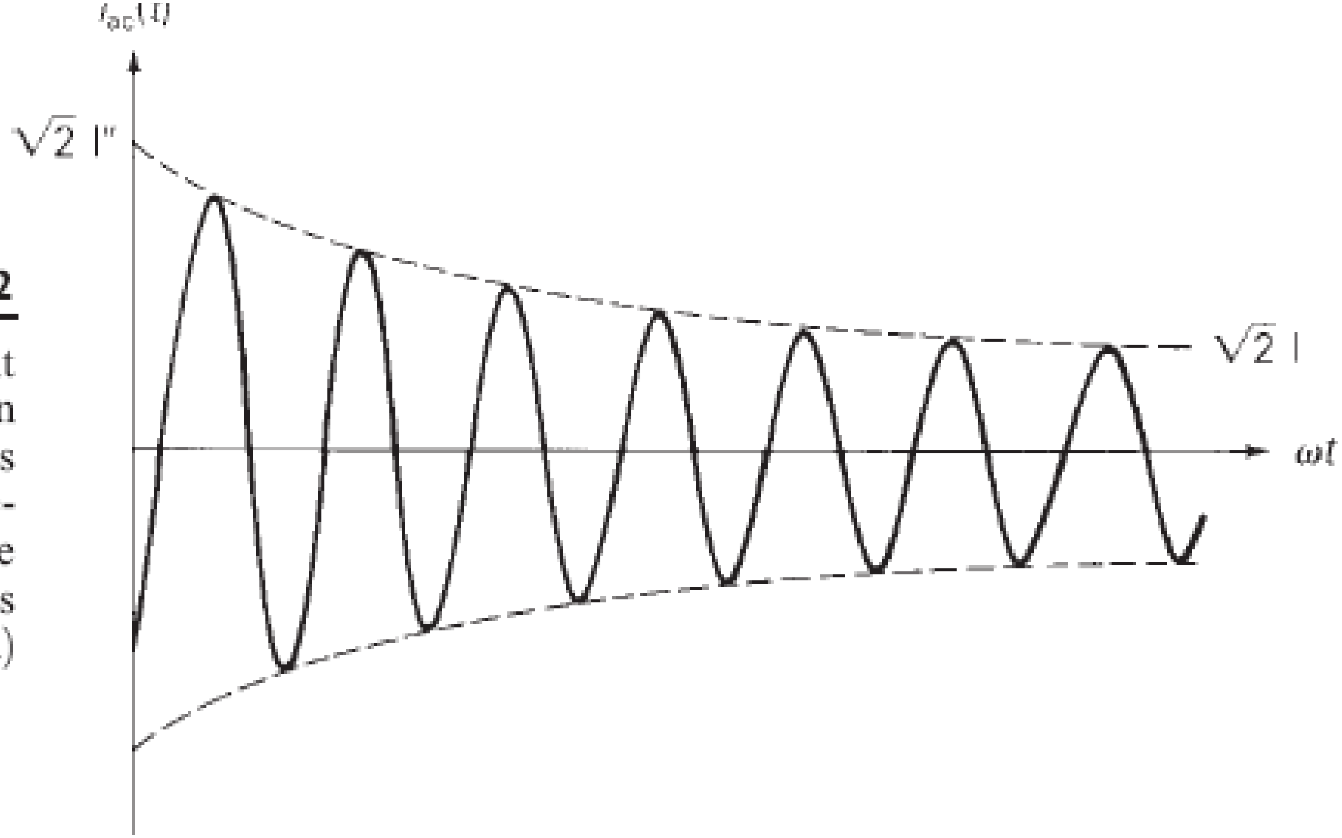
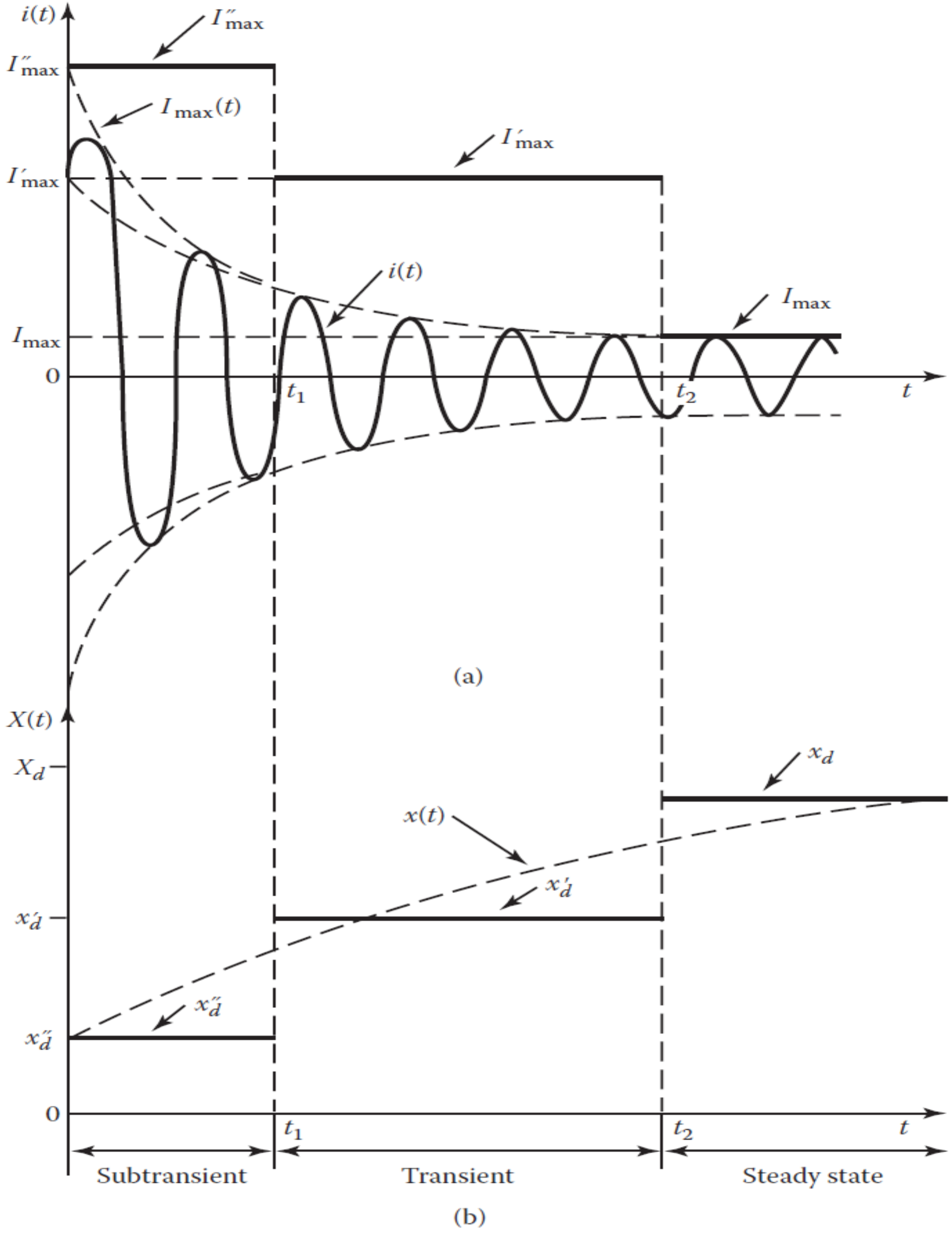


FIGURE 7.2

The ac fault current in one phase of an unloaded synchronous machine during a three-phase short circuit (the dc offset current is removed)



X''_d = subtransient reactance: determines the fault current during the first cycle after the fault occurs. In about 0.05−0.1 s, this reactance increases to

X'_d = transient reactance: determines the fault current after several cycles at 60 Hz. In about 0.2−2 s, it reactance increases to

X_d = X_s = synchronous reactance: determines the fault current after a steady-state condition is reached

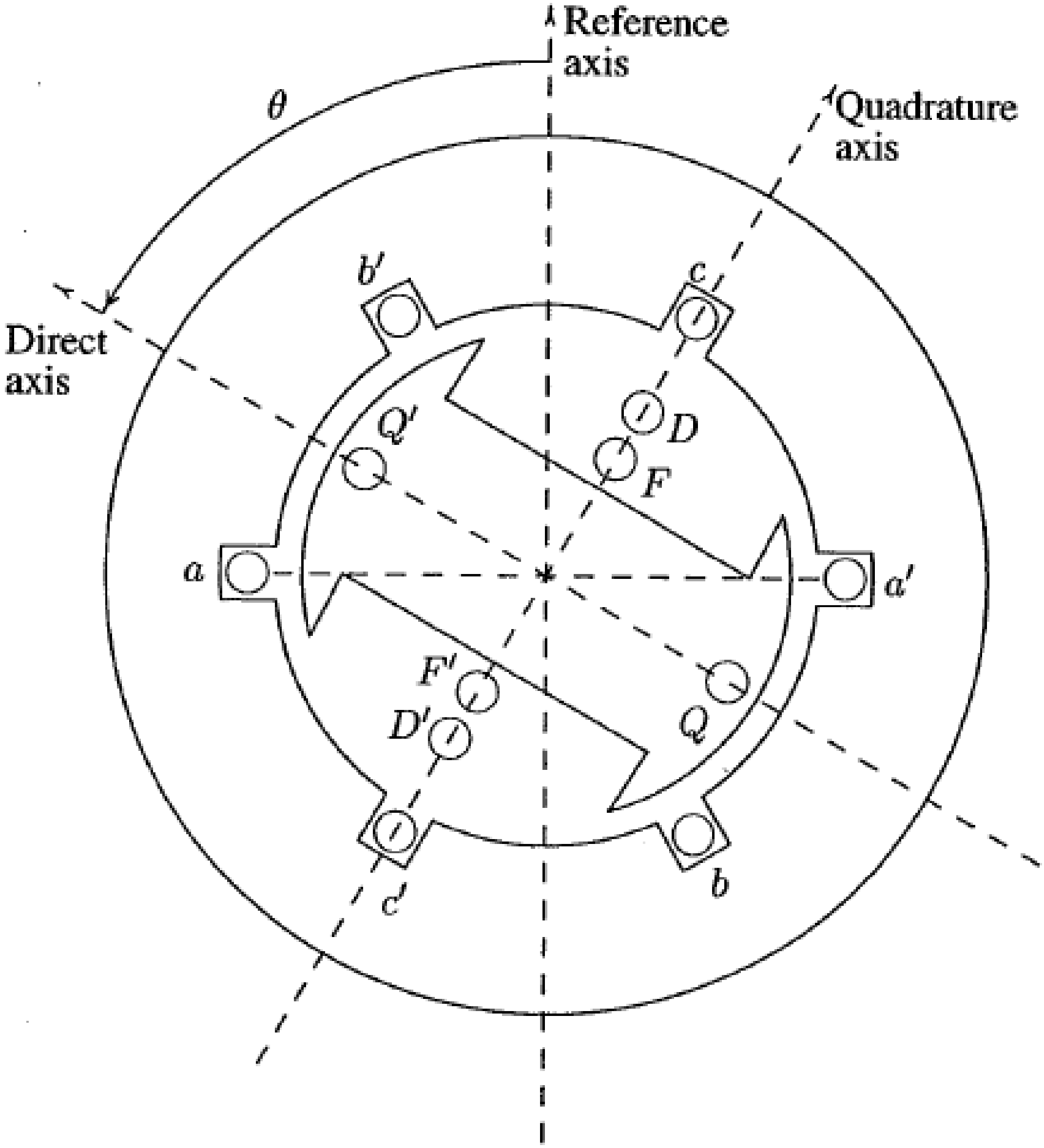


FIGURE 8.3
Schematic representation of a synchronous machine.

The ac fault current in a synchronous machine can be modeled by the series R–L circuit of Figure 7.1 if a time-varying inductance $L(t)$ or reactance $X(t) = \omega L(t)$ is employed. In standard machine theory texts [3, 4], the following reactances are defined:

$X_d'' =$ direct axis subtransient reactance

$X_d' =$ direct axis transient reactance

$X_d =$ direct axis synchronous reactance

where $X_d'' < X_d' < X_d$. The subscript d refers to the direct axis. There are similar quadrature axis reactances X_q'', X_q', and X_q [3, 4]. However, if the armature resistance is small, the quadrature axis reactances do not significantly affect the short-circuit current. Using the above direct axis reactances, the instantaneous ac fault current can be written as

$$i_{\text{ac}}(t) = \sqrt{2}E_g \left[\left(\frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-t/T_d''} + \left(\frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-t/T_d'} + \frac{1}{X_d} \right] \sin\left(\omega t + \alpha - \frac{\pi}{2} \right) \qquad (7.2.1)$$

where E_g is the rms line-to-neutral prefault terminal voltage of the unloaded synchronous machine. Armature resistance is neglected in (7.2.1). Note that at $t = 0$, when the fault occurs, the rms value of $i_{ac}(t)$ in (7.2.1) is

$$I_{ac}(0) = \frac{E_g}{X''_d} = I'' \tag{7.2.2}$$

which is called the rms *subtransient fault current*, I''. The duration of I'' is determined by the time constant T''_d, called the *direct axis short-circuit subtransient time constant.*

At a later time, when t is large compared to T''_d but small compared to the *direct axis short-circuit transient time constant* T'_d, the first exponential term in (7.2.1) has decayed almost to zero, but the second exponential has not decayed significantly. The rms ac fault current then equals the rms *transient fault current*, given by

$$I' = \frac{E_g}{X'_d} \tag{7.2.3}$$

When t is much larger than T_d', the rms ac fault current approaches its steady-state value, given by

$$I_{ac}(\infty) = \frac{E_g}{X_d} = I \tag{7.2.4}$$

Since the three-phase no-load voltages are displaced 120° from each other, the three-phase ac fault currents are also displaced 120° from each other. In addition to the ac fault current, each phase has a different dc offset. The maximum dc offset in any one phase, which occurs when $\alpha = 0$ in (7.2.1), is

$$i_{dcmax}(t) = \frac{\sqrt{2}E_g}{X_d''} e^{-t/T_A} = \sqrt{2}I'' e^{-t/T_A} \tag{7.2.5}$$

where T_A is called the *armature time constant*. Note that the magnitude of the maximum dc offset depends only on the rms subtransient fault current I''. The above synchronous machine short-circuit currents are summarized in Table 7.2.

TABLE 7.2

Short-circuit current—unloaded synchronous machine*

Component	Instantaneous Current (A)	rms Current (A)
Symmetrical (ac)	(7.2.1)	$I_{ac}(t) = E_g\left[\left(\frac{1}{X''_d} - \frac{1}{X'_d}\right)e^{-t/T''_d} + \left(\frac{1}{X'_d} - \frac{1}{X_d}\right)e^{-t/T'_d} + \frac{1}{X_d}\right]$
Subtransient		$I'' = E_g/X''_d$
Transient		$I' = E_g/X'_d$
Steady-state		$I = E_g/X_d$
Maximum dc offset	$i_{dc}(t) = \sqrt{2}I''e^{-t/T_A}$	
Asymmetrical (total)	$i(t) = i_{ac}(t) + i_{dc}(t)$	$I_{rms}(t) = \sqrt{I_{ac}(t)^2 + i_{dc}(t)^2}$ with maximum dc offset: $I_{rms}(t) = \sqrt{I_{ac}(t)^2 + [\sqrt{2}I''e^{-t/T_A}]^2}$

Machine reactances X''_d, X'_d, and X_d as well as time constants T''_d, T'_d, and T_A are usually provided by synchronous machine manufacturers. They can also be obtained from a three-phase short-circuit test, by analyzing an oscillogram such as that in Figure 7.2 [2]. Typical values of synchronous machine reactances and time constants are given in Appendix Table A.1.

TABLE A.1

Typical average values of synchronous-machine constants

Constant (units)	Type	Symbol	Turbo-Generator (solid rotor)	Water-Wheel Generator (with dampers)	Synchronous Condenser	Synchronous Motor
Reactances (per unit)	Synchronous	X_d	1.1	1.15	1.80	1.20
		X_q	1.08	0.75	1.15	0.90
	Transient	X'_d	0.23	0.37	0.40	0.35
		X'_q	0.23	0.75	1.15	0.90
	Subtransient	X''_d	0.12	0.24	0.25	0.30
		X''_q	0.15	0.34	0.30	0.40
	Negative-sequence	X_2	0.13	0.29	0.27	0.35
	Zero-sequence	X_0	0.05	0.11	0.09	0.16
Resistances (per unit)	Positive-sequence	R (dc)	0.003	0.012	0.008	0.01
		R (ac)	0.005	0.012	0.008	0.01
	Negative-sequence	R_2	0.035	0.10	0.05	0.06
Time constants (seconds)	Transient	T'_{d0}	5.6	5.6	9.0	6.0
		T'_d	1.1	1.8	2.0	1.4
	Subtransient	$T''_d = T''_q$	0.035	0.035	0.035	0.036
	Armature	T_a	0.16	0.15	0.17	0.15

(Adapted from E. W. Kimbark, *Power System Stability: Synchronous Machines* (New York: Dover Publications, 1956/1968), Chap. 12)

EXAMPLE 7.2 Three-phase short-circuit currents, unloaded synchronous generator

A 500-MVA 20-kV, 60-Hz synchronous generator with reactances $X''_d = 0.15$, $X'_d = 0.24$, $X_d = 1.1$ per unit and time constants $T''_d = 0.035$, $T'_d = 2.0$, $T_A = 0.20$ s is connected to a circuit breaker. The generator is operating at 5% above rated voltage and at no-load when a bolted three-phase short circuit occurs on the load side of the breaker. The breaker interrupts the fault 3 cycles after fault inception. Determine (a) the subtransient fault current in per-unit and kA rms; (b) maximum dc offset as a function of time; and (c) rms asymmetrical fault current, which the breaker interrupts, assuming maximum dc offset.

SOLUTION

a. The no-load voltage before the fault occurs is $E_g = 1.05$ per unit. From (7.2.2), the subtransient fault current that occurs in each of the three phases is

$$I'' = \frac{1.05}{0.15} = 7.0 \quad \text{per unit}$$

The generator base current is

$$I_{\text{base}} = \frac{S_{\text{rated}}}{\sqrt{3}V_{\text{rated}}} = \frac{500}{(\sqrt{3})(20)} = 14.43 \quad \text{kA}$$

The rms subtransient fault current in kA is the per-unit value multiplied by the base current:

$$I'' = (7.0)(14.43) = 101.0 \quad \text{kA}$$

b. From (7.2.5), the maximum dc offset that may occur in any one phase is

$$i_{\text{dcmax}}(t) = \sqrt{2}(101.0)e^{-t/0.20} = 142.9e^{-t/0.20} \quad \text{kA}$$

c. From (7.2.1), the rms ac fault current at $t = 3$ cycles $= 0.05$ s is

$$
\begin{aligned}
\mathrm{I}_{\mathrm{ac}}(0.05\ \mathrm{s}) &= 1.05\left[\left(\frac{1}{0.15} - \frac{1}{0.24}\right)e^{-0.05/0.035}\right. \\
&\qquad \left. + \left(\frac{1}{0.24} - \frac{1}{1.1}\right)e^{-0.05/2.0} + \frac{1}{1.1}\right] \\
&= 4.920 \quad \text{per unit} \\
&= (4.920)(14.43) = 71.01 \quad \mathrm{kA}
\end{aligned}
$$

Modifying (7.1.10) to account for the time-varying symmetrical component of fault current, we obtain

$$
\begin{aligned}
\mathrm{I}_{\mathrm{rms}}(0.05) &= \sqrt{[\mathrm{I}_{\mathrm{ac}}(0.05)]^2 + [\sqrt{2}\mathrm{I}'' e^{-t/\mathrm{T}_a}]^2} \\
&= \mathrm{I}_{\mathrm{ac}}(0.05)\sqrt{1 + 2\left[\frac{\mathrm{I}''}{\mathrm{I}_{\mathrm{ac}}(0.05)}\right]^2 e^{-2t/\mathrm{T}a}} \\
&= (71.01)\sqrt{1 + 2\left[\frac{101}{71.01}\right]^2 e^{-2(0.05)/0.20}} \\
&= (71.01)(1.8585) \\
&= 132 \quad \mathrm{kA}
\end{aligned}
$$

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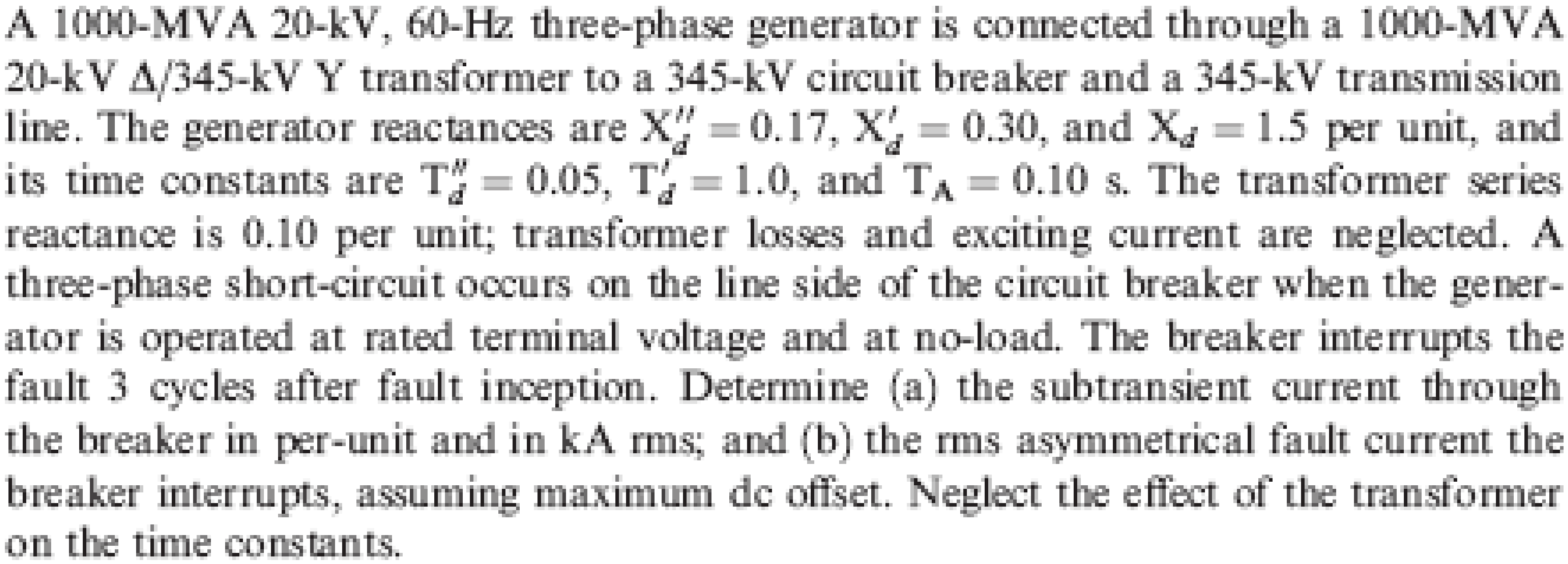
TUGAS 2



A 1000-MVA 20-kV, 60-Hz three-phase generator is connected through a 1000-MVA 20-kV Δ/345-kV Y transformer to a 345-kV circuit breaker and a 345-kV transmission line. The generator reactances are $X_d'' = 0.17$, $X_d' = 0.30$, and $X_d = 1.5$ per unit, and its time constants are $T_d'' = 0.05$, $T_d' = 1.0$, and $T_A = 0.10$ s. The transformer series reactance is 0.10 per unit; transformer losses and exciting current are neglected. A three-phase short-circuit occurs on the line side of the circuit breaker when the generator is operated at rated terminal voltage and at no-load. The breaker interrupts the fault 3 cycles after fault inception. Determine (a) the subtransient current through the breaker in per-unit and in kA rms; and (b) the rms asymmetrical fault current the breaker interrupts, assuming maximum dc offset. Neglect the effect of the transformer on the time constants.