Menganalisa gangguan 3 phase seimbang dengan menggunakan bus impedance matrix

$$Y_{bus}E^{(1)} = I^{(1)} \tag{7.4.1}$$

where Y_{bus} is the positive-sequence bus admittance matrix, $E^{(1)}$ is the vector of bus voltages, and $I^{(1)}$ is the vector of current sources. The superscript (1) denotes the first circuit. Solving (7.4.1),

$$Z_{bus}I^{(1)} = E^{(1)} \tag{7.4.2}$$

where

$$Z_{bus} = Y_{bus}^{-1} \tag{7.4.3}$$

Z_{bus}, the inverse of Y_{bus}, is called the positive-sequence *bus impedance matrix*. Both Z_{bus} and Y_{bus} are symmetric matrices.

Since the first circuit contains only one source, located at faulted bus n, the current source vector contains only one nonzero component, $I_n^{(1)} = -I''_{Fn}$. Also, the voltage at faulted bus n in the first circuit is $E_n^{(1)} = -V_F$. Rewriting (7.4.2),

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} & \cdots & Z_{2N} \\ \vdots & & & & & \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} & \cdots & Z_{nN} \\ \vdots & & & & & \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nn} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -I''_{Fn} \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} E_1^{(1)} \\ E_2^{(1)} \\ \vdots \\ -V_F \\ \vdots \\ E_N^{(1)} \end{bmatrix} \tag{7.4.4}$$

The minus sign associated with the current source in (7.4.4) indicates that the current injected into bus n is the negative of I''_{Fn}, since I''_{Fn} flows away from bus n to the neutral. From (7.4.4), the subtransient fault current is

$$I''_{Fn} = \frac{V_F}{Z_{nn}} \tag{7.4.5}$$

Also from (7.4.4) and (7.4.5), the voltage at any bus k in the first circuit is

$$E_k^{(1)} = Z_{kn}(-I''_{Fn}) = \frac{-Z_{kn}}{Z_{nn}} V_F \tag{7.4.6}$$

The second circuit represents the prefault conditions. Neglecting prefault load current, all voltages throughout the second circuit are equal to the prefault voltage; that is, $E_k^{(2)} = V_F$ for each bus k. Applying superposition,

$$E_k = E_k^{(1)} + E_k^{(2)} = \frac{-Z_{kn}}{Z_{nn}} V_F + V_F$$

$$= \left(1 - \frac{Z_{kn}}{Z_{nn}}\right) V_F \qquad k = 1, 2, \ldots, N \qquad (7.4.7)$$

EXAMPLE 7.4 Using Z_{bus} to compute three-phase short-circuit currents in a power system

Faults at bus 1 and 2 in Figure 7.3 are of interest. The prefault voltage is 1.05 per unit and prefault load current is neglected. (a) Determine the 2×2 positive-sequence bus impedance matrix. (b) For a bolted three-phase short circuit at bus 1, use $\boldsymbol{Z}_{\text{bus}}$ to calculate the subtransient fault current and the contribution to the fault current from the transmission line. (c) Repeat part (b) for a bolted three-phase short circuit at bus 2.

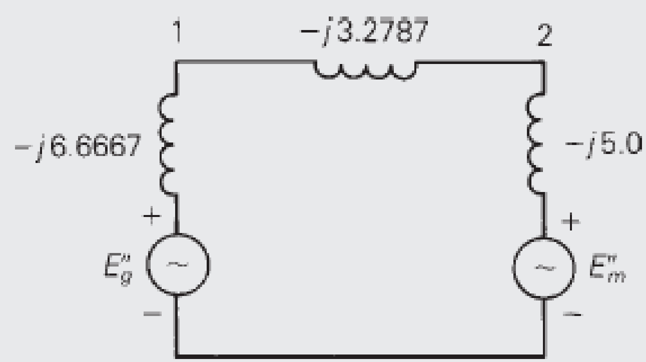

FIGURE 7.5

Circuit of Figure 7.4(a) showing per-unit admittance values

SOLUTION

a. The circuit of Figure 7.4(a) is redrawn in Figure 7.5 showing per-unit admittance rather than per-unit impedance values. Neglecting prefault load current, $E''_g = E''_m = V_F = 1.05\underline{/0^\circ}$ per unit. From Figure 7.5, the positive-sequence bus admittance matrix is

$$\boldsymbol{Y}_{\text{bus}} = -j\begin{bmatrix} 9.9454 & -3.2787 \\ -3.2787 & 8.2787 \end{bmatrix} \quad \text{per unit}$$

Inverting $\boldsymbol{Y}_{\text{bus}}$,

$$\boldsymbol{Z}_{\text{bus}} = \boldsymbol{Y}_{\text{bus}}^{-1} = +j\begin{bmatrix} 0.11565 & 0.04580 \\ 0.04580 & 0.13893 \end{bmatrix} \quad \text{per unit}$$

b. Using (7.4.5) the subtransient fault current at bus 1 is

$$I''_{F1} = \frac{V_F}{Z_{11}} = \frac{1.05\underline{/0^\circ}}{j0.11565} = -j9.079 \quad \text{per unit}$$

which agrees with the result in Example 7.3, part (a). The voltages at buses 1 and 2 during the fault are, from (7.4.7),

$$E_1 = \left(1 - \frac{Z_{11}}{Z_{11}}\right) V_F = 0$$

$$E_2 = \left(1 - \frac{Z_{21}}{Z_{11}}\right) V_F = \left(1 - \frac{j0.04580}{j0.11565}\right) 1.05\underline{/0^\circ} = 0.6342\underline{/0^\circ}$$

The current to the fault from the transmission line is obtained from the voltage drop from bus 2 to 1 divided by the impedance of the line and transformers T_1 and T_2:

$$I_{21} = \frac{E_2 - E_1}{j(X_{\text{line}} + X_{T1} + X_{T2})} = \frac{0.6342 - 0}{j0.3050} = -j2.079 \quad \text{per unit}$$

which agrees with the motor current calculated in Example 7.3, part (b), where prefault load current is neglected.

c. Using (7.4.5), the subtransient fault current at bus 2 is

$$I''_{F2} = \frac{V_F}{Z_{22}} = \frac{1.05\underline{/0^\circ}}{j0.13893} = -j7.558 \quad \text{per unit}$$

and from (7.4.7),

$$E_1 = \left(1 - \frac{Z_{12}}{Z_{22}}\right) V_F = \left(1 - \frac{j0.04580}{j0.13893}\right) 1.05\underline{/0^\circ} = 0.7039\underline{/0^\circ}$$

$$E_2 = \left(1 - \frac{Z_{22}}{Z_{22}}\right) V_F = 0$$

The current to the fault from the transmission line is

$$I_{12} = \frac{E_1 - E_2}{j(X_{\text{line}} + X_{T1} + X_{T2})} = \frac{0.7039 - 0}{j0.3050} = -j2.308 \quad \text{per unit}$$

■

ALGORITHM FOR BUILDING Z_{BUS} MATRIX

Z_{BUS} building algorithm is a step-by-step procedure which proceeds branch by branch. Main advantage of this method is that, any modification of the network elements does not require complete rebuilding of $Z_{BUS-matrix}$. Details of Z_{BUS} formulation is given below:

8.6.1 Type-1 Modification

In this case, branch impedance Z_b is added from a new bus to the reference bus. That is a new bus is added to the network and dimension of Z_{BUS} goes up by one.

Notations: i, j – old buses

r – reference bus

K – new bus

Figure 8.22 shows a passive linear n-bus power system network. In Fig. 8.22, an impedance Z_b is added between new bus K and the reference bus r.

From Fig. 8.22,

$$V_K = Z_b\, I_K$$

$$Z_{Ki} = Z_{iK} = 0; \quad \text{for } i = 1, 2, ..., n.$$

$$\therefore \qquad Z_{KK} = Z_b$$

Therefore,

$$Z_{BUS}^{new} = \left[\begin{array}{ccc|c} & Z_{BUS}^{old} & & \begin{matrix} 0 \\ \vdots \\ \vdots \\ 0 \end{matrix} \\ \hline 0 & \ldots\ldots & 0 & Z_b \end{array}\right] \qquad ...(8.22)$$

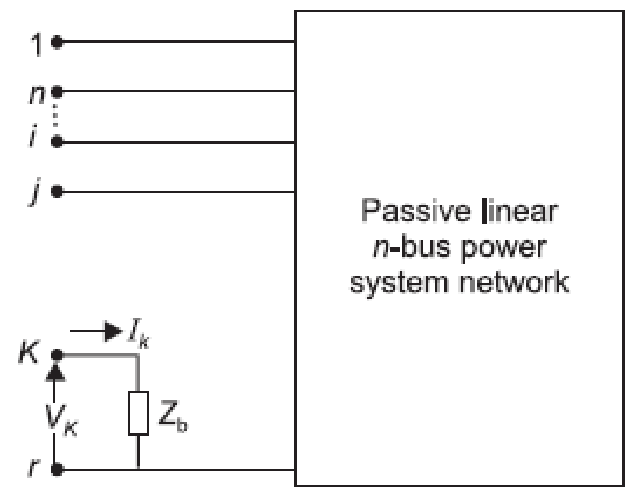

Fig. 8.22: Type-1 Modification.

where Z_{BUS}^{old} is bus impedance matrix before adding a new branch.

8.6.2 Type-2 Modification

In this case branch impedance Z_b is added from a new bus K to the old bus j as shown in Fig. 8.23.

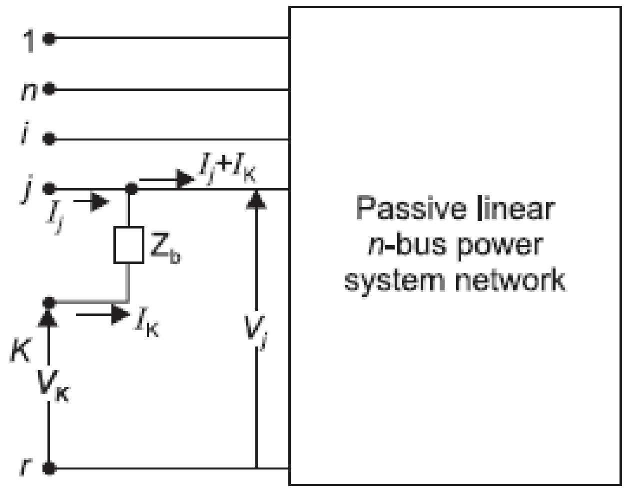

Fig. 8.23: Type-2 Modification.

From Fig. 8.23, we have

$$V_K = V_j + Z_b I_K$$

$$\therefore \quad V_K = Z_{j1} I_1 + Z_{j2} I_2 + \ldots\ldots + Z_{jj} (I_j + I_K) + \ldots\ldots + Z_{jn} I_n + Z_b I_K$$

$$\therefore \quad V_K = Z_{j1} I_1 + Z_{j2} I_2 + \ldots\ldots + Z_{jj} I_j + \ldots\ldots + Z_{jn} I_n + (Z_{jj} + Z_b) I_K \qquad \ldots(8.23)$$

Hence,

$$Z_{BUS}^{new} = \left[\begin{array}{cccc|c} & & & & Z_{1j} \\ & Z_{BUS}^{old} & & & Z_{2j} \\ & & & & \vdots \\ & & & & Z_{nj} \\ \hline Z_{j1} & Z_{j2} & \cdots & Z_{jn} & Z_{jj} + Z_b \end{array}\right] \qquad \ldots(8.24)$$

8.6.3 Type-3 Modification

In this case, an old bus-j is connected to the reference bus-r and the impedance between these two bus is Z_b as shown in Fig. 8.24.

Referring to Fig. 8.23, if bus K is connected to reference bus r, $V_K = 0$.

Thus

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ \hline 0 \end{bmatrix} = \left[\begin{array}{cccc|c} & & & & Z_{1j} \\ & Z_{BUS}^{old} & & & Z_{2j} \\ & & & & \vdots \\ & & & & Z_{nj} \\ \hline Z_{j1} & Z_{j2} & \cdots & Z_{jn} & Z_{jj} + Z_b \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ \hline I_K \end{bmatrix} \quad \text{...(8.25)}$$

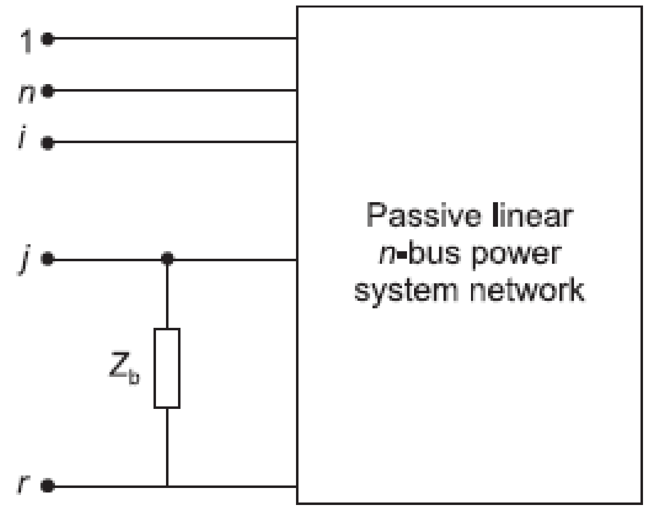

Fig. 8.24: Type–3 modification.

From equ. (8.25), we get,

$$O = Z_{j1} I_1 + Z_{j2} I_2 + ... + Z_{jn} I_n + (Z_{jj} + Z_b) I_K$$

$$\therefore \qquad I_K = \frac{-1}{(Z_{jj} + Z_b)} (Z_{j1} I_1 + Z_{j2} I_2 + ... + Z_{jn} I_n) \quad \text{...(8.26)}$$

Expression of voltage for i-th bus can be written as:

$$V_i = Z_{i1} I_1 + Z_{i2} I_2 + ... + Z_{in} I_n + Z_{ij} I_K \quad \text{...(8.27)}$$

From equations (8.27) and (8.26), we get

$$\therefore \qquad V_i = \left[Z_{i1} - \frac{Z_{ij}\,Z_{j1}}{Z_{jj} + Z_b}\right] I_1 + \left[Z_{i2} - \frac{Z_{ij}\,Z_{j2}}{Z_{jj} + Z_b}\right] I_2$$

$$+ \ldots\ldots + \left[Z_{in} - \frac{Z_{ij}\,Z_{jn}}{Z_{jj} + Z_b}\right] I_n \qquad \ldots(8.28)$$

By inspection, Z_{BUS}^{new} can easily be written from equation (8.28),

$$Z_{BUS}^{new} = Z_{BUS}^{old} - \frac{1}{\left(Z_{jj} + Z_b\right)} \begin{bmatrix} Z_{1j} \\ Z_{2j} \\ \vdots \\ Z_{ij} \\ \vdots \\ Z_{nj} \end{bmatrix} \begin{bmatrix} Z_{j1} & Z_{j2} & \cdots & Z_{jn} \end{bmatrix} \qquad \ldots(8.29)$$

8.6.4 Type-4 Modification

In this case, two old buses are connected and impedance between these buses is Z_b as shown in Fig. 8.25.

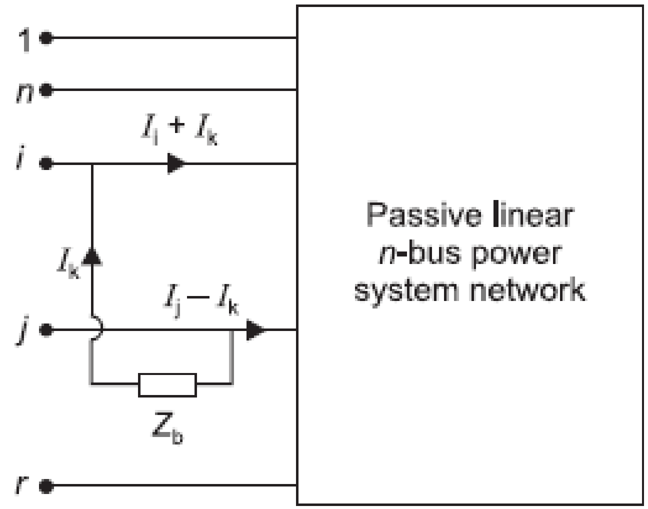

Fig. 8.25: Type-4 modification.

From Fig. 8.25, we can write,

$$V_i = Z_{i1} I_1 + Z_{i2} I_2 + ... + Z_{ii} (I_i + I_k) + Z_{ij} (I_j - I_k) + ... + Z_{in} I_n \quad ...(8.30)$$

Also

$$V_j = Z_b I_k + V_i \quad ...(8.31)$$

$$V_j = Z_{j1} I_1 + Z_{j2} I_2 + ... + Z_{ji} (I_i + I_k) + Z_{jj} (I_j - I_k) + ... + Z_{jn} I_n \quad ...(8.32)$$

From equations (8.30), (8.31) and (8.32), we get

$$Z_{j1} I_1 + Z_{j2} I_2 + ... + Z_{ji} (I_i + I_k) + Z_{jj} (I_j - I_k) + ... + Z_{jn} I_n = Z_b I_k + Z_{i1} I_1$$
$$+ Z_{i2} I_2 + ... + Z_{ii} (I_i + I_k) + Z_{ij} (I_j - I_k) + ... + Z_{in} I_n$$

$$\therefore \quad 0 = (Z_{i1} - Z_{j1}) I_1 + (Z_{i2} - Z_{j2}) I_2 + ... + (Z_{ii} - Z_{ji}) I_i + (Z_{ij} - Z_{jj}) I_j + ...$$
$$+ (Z_{in} - Z_{jn}) I_n + (Z_b + Z_{ii} + Z_{jj} - Z_{ij} - Z_{ji}) I_k \quad ...(8.33)$$

Note that $Z_{ij} = Z_{ji}$ and coefficient of I_k is $(Z_b + Z_{ii} + Z_{jj} - 2 Z_{ij})$

or

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ \hline 0 \end{bmatrix} = \left[\begin{array}{ccc|c} & & & (Z_{1i} - Z_{1j}) \\ & & & (Z_{2i} - Z_{2j}) \\ & Z_{BUS}^{old} & & \vdots \\ & & & (Z_{ni} - Z_{nj}) \\ \hline (Z_{i1} - Z_{j1}) & \cdots & (Z_{in} - Z_{jn}) & (Z_b + Z_{ii} + Z_{jj} - 2Z_{ij}) \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ \hline I_K \end{bmatrix} \quad ...(8.34)$$

Eliminating I_k in equation (8.34) and following the same procedure for Type-2 modification, we get,

$$Z_{BUS}^{new} = Z_{BUS}^{old} - \frac{1}{(Z_b + Z_{ii} + Z_{jj} - 2 Z_{ij})} \begin{bmatrix} (Z_{1i} - Z_{1j}) \\ (Z_{2i} - Z_{2j}) \\ \vdots \\ (Z_{ni} - Z_{nj}) \end{bmatrix} \times$$
$$[(Z_{i1} - Z_{j1}) \ (Z_{i2} - Z_{j2}) \ ... \ (Z_{in} - Z_{jn})] \quad ... (8.35)$$

With the use of above mentioned four modifications bus impedance matrix can be formulated by a step-by-step technique considering one branch at a time.

Example 8.19 *Fig. 8.26 shows a three bus network. Obtain impedance matrix Z_{BUS}.*

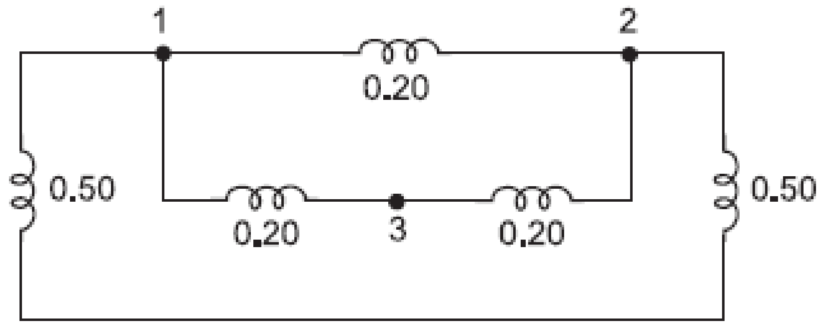

Fig. 8.26: Three bus network.

Solution:

Step-1: Add branch $Z_{1r} = 0.50$ (from new bus 1 to reference bus *r*)

$$\therefore \quad Z_{BUS} = [0.50] \quad \text{... (i)}$$

Step-2: Type-2 modification. That is add branch $Z_{21} = 0.20$ (from new bus 2 to old bus 1)

$$\therefore \quad Z_{BUS} = \begin{matrix} 1 \\ 2 \end{matrix}\begin{bmatrix} 0.50 & 0.50 \\ 0.50 & 0.70 \end{bmatrix} \quad \text{...(ii)}$$

Step-3: Add branch $Z_{13} = 0.20$ from new bus 3 to old bus 1. This is type-2 modification.

$$\therefore \quad Z_{BUS} = \begin{bmatrix} 0.50 & 0.50 & 0.50 \\ 0.50 & 0.70 & 0.50 \\ 0.50 & 0.50 & 0.70 \end{bmatrix}$$

Step-4: Add branch Z_{2r} from old bus 2 to reference bus r. This is type-3 modification.

$$\therefore \qquad Z_{BUS} = \begin{bmatrix} 0.50 & 0.50 & 0.50 \\ 0.50 & 0.70 & 0.50 \\ 0.50 & 0.50 & 0.70 \end{bmatrix} - \frac{1}{(0.7 + 0.50)} \begin{bmatrix} 0.50 \\ 0.70 \\ 0.50 \end{bmatrix} [0.5 \quad 0.7 \quad 0.5]$$

$$\therefore \qquad Z_{BUS} = \begin{bmatrix} 0.2916 & 0.2084 & 0.2916 \\ 0.2084 & 0.2916 & 0.2084 \\ 0.2916 & 0.2084 & 0.4916 \end{bmatrix}$$

Step-5: Add branch $Z_{23} = 0.20$ from old bus 2 to old bus 3. This is type-4 modification.

$$\therefore \qquad Z_{BUS} = \begin{bmatrix} 0.2916 & 0.2084 & 0.2916 \\ 0.2084 & 0.2916 & 0.2084 \\ 0.2916 & 0.2084 & 0.4916 \end{bmatrix}$$

$$- \frac{1}{(0.20 + 0.2916 + 0.4916 - 2 \times 0.2084)} \begin{bmatrix} -0.0832 \\ 0.0832 \\ -0.2832 \end{bmatrix} [-0.0832 \quad 0.0832 \quad -0.2832]$$

$$\therefore \qquad Z_{BUS} = \begin{bmatrix} 0.2793 & 0.2206 & 0.2500 \\ 0.2206 & 0.2793 & 0.2500 \\ 0.2500 & 0.2500 & 0.3500 \end{bmatrix}$$

TUGAS 4

A single-line diagram of a four-bus system is shown in Figure 7.20, for which Z_{BUS} is given below:

$$Z_{BUS} = j\begin{bmatrix} 0.25 & 0.2 & 0.16 & 0.14 \\ 0.2 & 0.23 & 0.15 & 0.151 \\ 0.16 & 0.15 & 0.196 & 0.1 \\ 0.14 & 0.151 & 0.1 & 0.195 \end{bmatrix} \text{ per unit}$$

Let a three-phase fault occur at bus 2 of the network.

(a) Calculate the initial symmetrical RMS current in the fault.

(b) Determine the voltages during the fault at buses 1, 3, and 4.

(c) Compute the fault currents contributed to bus 2 by the adjacent unfaulted buses 1, 3, and 4.

(d) Find the current flow in the line from bus 3 to bus 1. Assume the prefault voltage V_f at bus 2 to be $1\underline{/0^\circ}$ pu, and neglect all prefault currents.

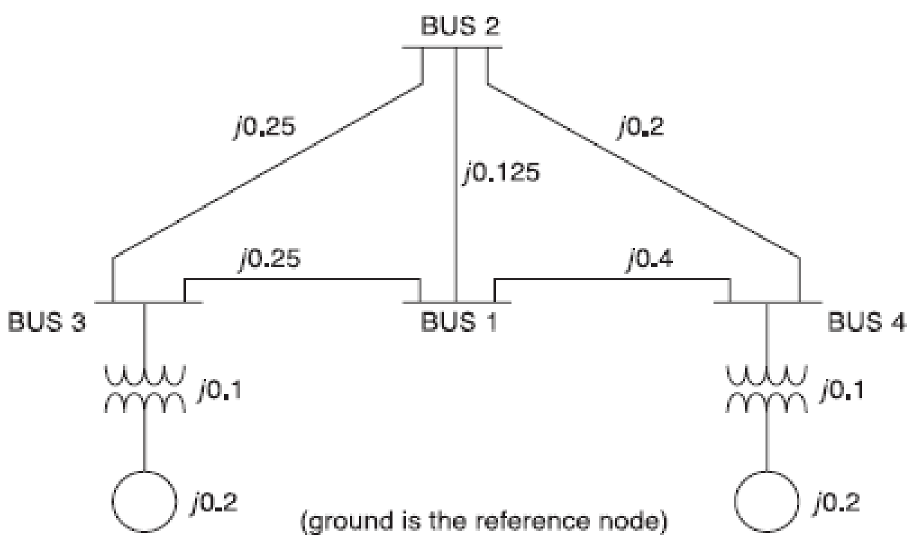

FIGURE 7.20

Single-line diagram for Problem 7.23