KOMPONEN-KOMPONEN SIMETRI

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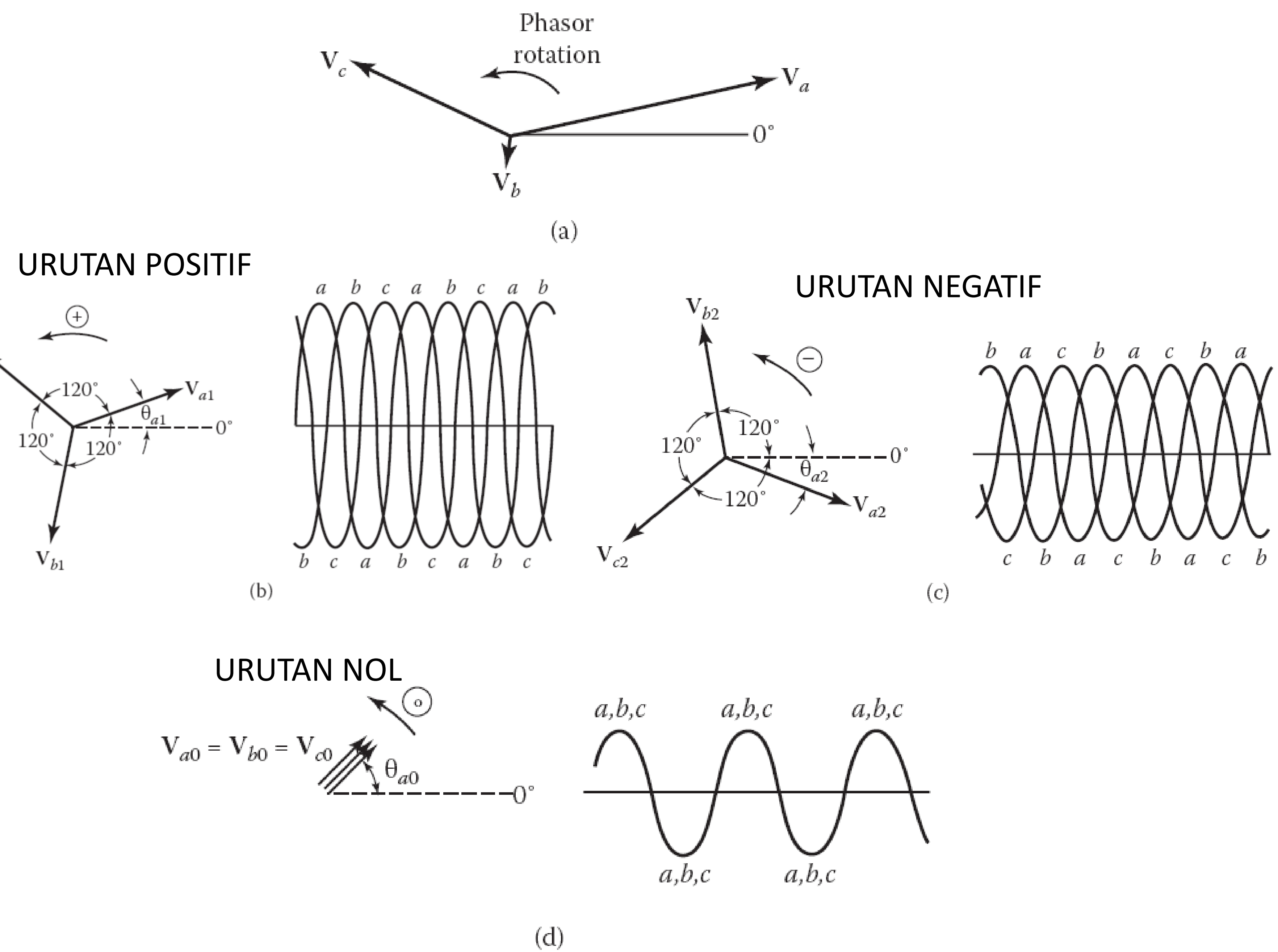
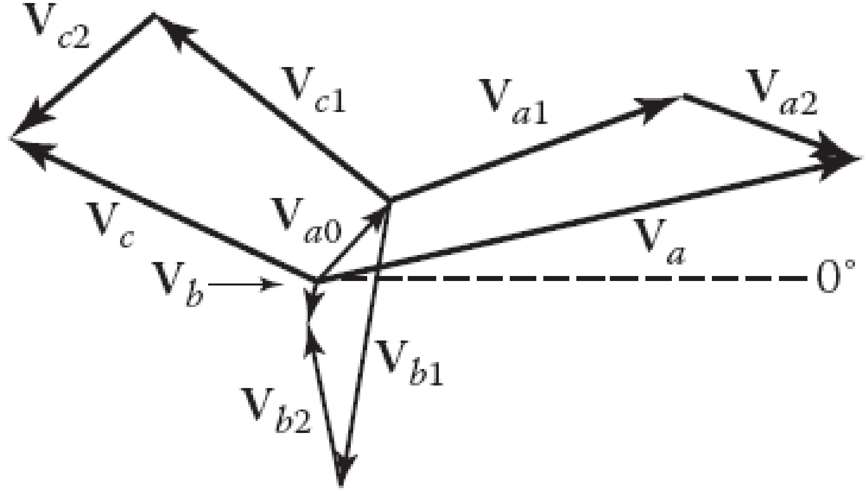
- Pada tahun 1918, C.L. Fortescue, suatu sistem tak seimbang yang terdiri dari *n* phasor-phasor yang berhubungan dapat diuraikan menjadi *n* buah sistem dengan phasor-phasor seimbang yang dinamakan **komponen-komponen simetri**

1. *Zero-sequence* components, consisting of three phasors with equal magnitudes and with zero phase displacement,
2. *Positive-sequence* components, consisting of three phasors with equal magnitudes, ±120° phase displacement, and positive sequence,
3. *Negative-sequence* components, consisting of three phasors with equal magnitudes, ±120° phase displacement, and negative sequence,



Phasor rotation
V c
V a
0°
V b
(a)
URUTAN POSITIF
+
V c1
120°
V a1
θ a1
0°
120°
120°
a b c a b c a b
b c a b c a b c
(b)
URUTAN NEGATIF
V b2
−
120°
120°
0°
θ a2
120°
V a2
V c2
b a c b a c b a
c b a c b a c b
(c)
URUTAN NOL
o
V a0 = V b0 = V c0
θ a0
0°
a,b,c a,b,c a,b,c
a,b,c a,b,c
(d)

PENJUMLAHAN SECARA GRAFIS KOMPONEN-KOMPONEN SIMETRI UNTUK MENDAPATKAN TIGA PHASOR –PHASOR TAK SEIMBANG



(e)

$$\mathbf{V}_a = \mathbf{V}_{a1} + \mathbf{V}_{a2} + \mathbf{V}_{a0}$$

$$\mathbf{V}_b = \mathbf{V}_{b1} + \mathbf{V}_{b2} + \mathbf{V}_{b0}$$

$$\mathbf{V}_c = \mathbf{V}_{c1} + \mathbf{V}_{c2} + \mathbf{V}_{c0}$$

OPERATOR a

$$
\begin{aligned}
\mathbf{a} &= 1\angle 120^\circ \\
&= 1e^{j\left(\frac{2\pi}{3}\right)} \\
&= 1(\cos 120^\circ + j\sin 120^\circ) \\
&= -0.5 + j0.866
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{a}^2 &= \mathbf{a}\times\mathbf{a} \\
&= (1\angle 120^\circ)(1\angle 120^\circ) = 1\angle 240^\circ = 1\angle -120^\circ \\
\mathbf{a}^3 &= \mathbf{a}^2\times\mathbf{a} \\
&= (1\angle 240^\circ)(1\angle 120^\circ) = 1\angle 360^\circ = 1\angle 0^\circ \\
\mathbf{a}^4 &= \mathbf{a}^3\times\mathbf{a} \\
&= (1\angle 0^\circ)(1\angle 120^\circ) = 1\angle 120^\circ = \mathbf{a} \\
\mathbf{a}^5 &= \mathbf{a}^3\times\mathbf{a}^2 \\
&= (1\angle 0^\circ)(1\angle 240^\circ) = 1\angle 240^\circ = \mathbf{a}^2 \\
\mathbf{a}^6 &= \mathbf{a}^3\times\mathbf{a}^3 \\
&= (1\angle 0^\circ)(1\angle 0^\circ) = 1\angle 0^\circ = \mathbf{a}^3 \\
&\vdots \\
\mathbf{a}^{n+3} &= \mathbf{a}^n\times\mathbf{a}^3 = \mathbf{a}^n
\end{aligned}
$$

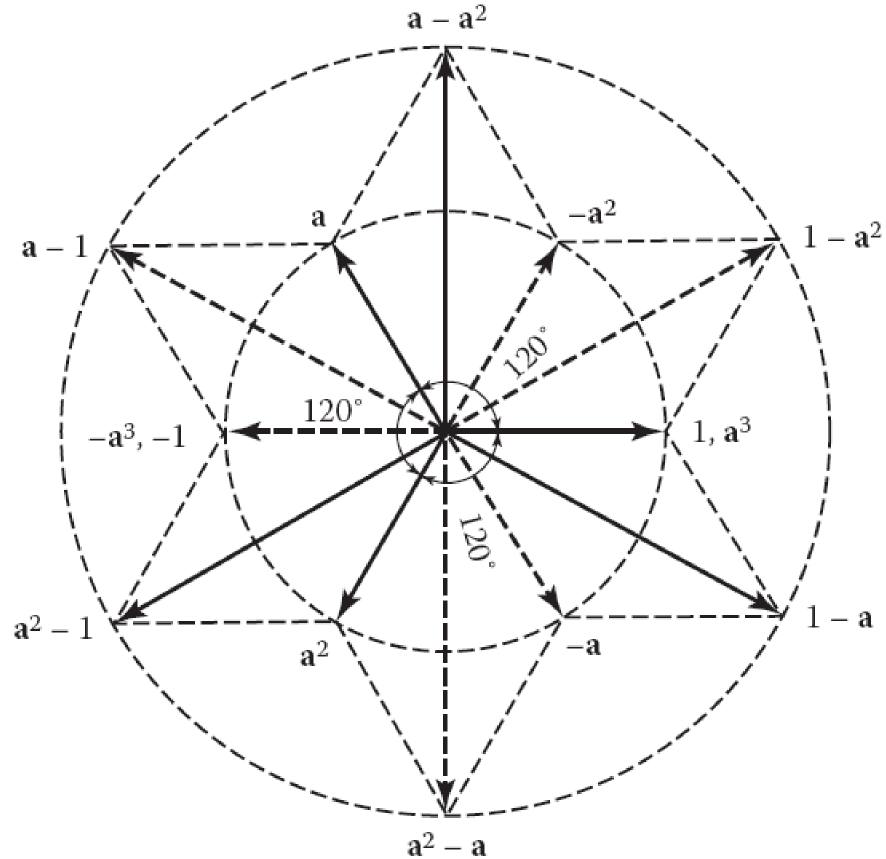


TABLE 5.1
Powers and Functions of Operator a

Power or Function	In Polar Form	In Rectangular Form
$\mathbf{a}$	$1\angle 120°$	$-0.5 + j0.866$
$\mathbf{a}^2$	$1\angle 240° = 1\angle -120°$	$-0.5 - j0.866$
$\mathbf{a}^3$	$1\angle 360° = 1\angle 0°$	$1.0 + j0.0$
$\mathbf{a}^4$	$1\angle 120°$	$-0.5 + j0.866$
$1 + \mathbf{a} = -\mathbf{a}^2$	$1\angle 60°$	$0.5 + j0.866$
$1 - \mathbf{a}$	$\sqrt{3}\angle -30°$	$1.5 - j0.866$
$1 + \mathbf{a}^2 = -\mathbf{a}$	$1\angle -60°$	$0.5 - j0.866$
$1 - \mathbf{a}^2$	$\sqrt{3}\angle 30°$	$1.5 + j0.866$
$\mathbf{a} - 1$	$\sqrt{3}\angle 150°$	$-1.5 + j0.866$
$\mathbf{a} + \mathbf{a}^2$	$1\angle 180°$	$-1.0 + j0.0$
$\mathbf{a} - \mathbf{a}^2$	$\sqrt{3}\angle 90°$	$0.0 + j1.732$
$\mathbf{a}^2 - \mathbf{a}$	$\sqrt{3}\angle -90°$	$0.0 - j1.732$
$\mathbf{a}^2 - 1$	$\sqrt{3}\angle -150°$	$-1.5 - j0.866$
$1 + \mathbf{a} + \mathbf{a}^2$	$0\angle 0°$	$0.0 + j0.0$

DENGAN MENGGUNAKAN OPERATOR a

$$\mathbf{V}_{b1} = \mathbf{a}^2\mathbf{V}_{a1}$$

$$\mathbf{V}_{c1} = \mathbf{a}\mathbf{V}_{a1}$$

$$\mathbf{V}_{b2} = \mathbf{a}\mathbf{V}_{a2}$$

$$\mathbf{V}_{c2} = \mathbf{a}^2\mathbf{V}_{a2}$$

$$\mathbf{V}_{b0} = \mathrm{V}_{c0} = \mathbf{V}_{a0}$$

$$\mathbf{V}_{a} = \mathbf{V}_{a1} + \mathbf{V}_{a2} + \mathbf{V}_{a0}$$

$$\mathbf{V}_{b} = \mathbf{a}^2\mathbf{V}_{a1} + \mathbf{a}\mathbf{V}_{a2} + \mathbf{V}_{a0}$$

$$\mathbf{V}_{c} = \mathbf{a}\mathbf{V}_{a1} + \mathbf{a}^2\mathbf{V}_{a2} + \mathbf{V}_{a0}$$

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix}$$

$$[\mathbf{A}] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \Rightarrow [\mathbf{A}]^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}$$

INVERS

$$[\mathbf{V}_{012}] = [\mathbf{A}]^{-1}\,[\mathbf{V}_{abc}]$$

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} \Rightarrow \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix}$$

$$[\mathbf{I}_{012}] = [\mathbf{A}]^{-1}\,[\mathbf{I}_{abc}]$$

$$\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} \Rightarrow \begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix}$$

EXAMPLE 5.1

Determine the symmetrical components for the phase voltages of $\mathbf{V}_a = 7.3\angle 12.5$, $\mathbf{V}_b = 0.4\angle{-100°}$, and $\mathbf{V}_c = 4.4\angle 154°$ V

$$\mathbf{V}_{a0} = \frac{1}{3}\left(\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c\right)$$

$$= \frac{1}{3}(7.3\angle 12.5° + 0.4\angle -100° + 4.4\angle 154°)$$

$$= 1.47\angle 45.1° \text{ V}$$

$$\mathbf{V}_{a1} = \frac{1}{3}\left(\mathbf{V}_a + \mathbf{a}\mathbf{V}_b + \mathbf{a}^2\mathbf{V}_c\right)$$

$$= \frac{1}{3}\left[7.3\angle 12.5° + (1\angle 120°)(0.4\angle -100°) + (1\angle 240°)(4.4\angle 154°)\right]$$

$$= 3.97\angle 20.5° \text{ V}$$

$$\mathbf{V}_{a2} = \frac{1}{3}\left(\mathbf{V}_a + \mathbf{a}^2\mathbf{V}_b + \mathbf{a}\mathbf{V}_c\right)$$

$$= 3\left[7.3\angle 12.5° + (1\angle 240°)(0.4\angle -100°) + (1\angle 20°)(4.4\angle 154°)\right]$$

$$= 2.52\angle -19.7° \text{ V}$$

$$\mathbf{V}_{b0} = \mathbf{V}_{a0} = 1.47\angle 45.1^\circ \text{ V}$$

$$\mathbf{V}_{c0} = \mathbf{V}_{a0} = 1.47\angle 45.1^\circ \text{ V}$$

$$\mathbf{V}_{b1} = \mathbf{a}^2\mathbf{V}_{a1} = (1\angle 240^\circ)(3.97\angle 20.5^\circ) = 3.97\angle 260.5^\circ \text{ V}$$

$$\mathbf{V}_{c1} = \mathbf{a}\mathbf{V}_{a1} = (1\angle 120^\circ)(3.97\angle 20.5^\circ) = 3.97\angle 140.5^\circ \text{ V}$$

$$\mathbf{V}_{b2} = \mathbf{a}\mathbf{V}_{a2} = (1\angle 120^\circ)(2.52\angle -19.7^\circ) = 2.52\angle 100.3^\circ \text{ V}$$

$$\mathbf{V}_{c2} = \mathbf{a}^2\mathbf{V}_{a2} = (1\angle 240^\circ)(2.52\angle -19.7^\circ) = 2.52\angle 220.3^\circ \text{ V}$$

DAYA DENGAN KOMPONEN-KOMPONEN SIMETRIS

$$\mathbf{S}_{3\phi} = P_{3\phi} + jQ_{3\phi}$$
$$= \mathbf{S}_a + \mathbf{S}_b + \mathbf{S}_c$$
$$= \mathbf{V}_a\mathbf{I}_a^* + \mathbf{V}_b\mathbf{I}_b^* + \mathbf{V}_c\mathbf{I}_c^*$$

MATRIK

$$\mathbf{S}_{3\phi} = \begin{bmatrix} \mathbf{V}_a & \mathbf{V}_b & \mathbf{V}_c \end{bmatrix} \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix}^*$$
$$= \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix}^t \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix}^*$$

$$\mathbf{S}_{3\phi} = [\mathbf{V}_{abc}]^t \, [\mathbf{I}_{abc}]^*$$

$$[\mathbf{V}_{abc}] = [\mathbf{A}] \, [\mathbf{V}_{012}]$$

$$[\mathbf{I}_{abc}] = [\mathbf{A}] \, [\mathbf{I}_{012}]$$

$$[\mathbf{V}_{abc}]^t = [\mathbf{V}_{012}]^t\,[\mathbf{A}]^t$$

$$[\mathbf{I}_{abc}]^* = [\mathbf{A}]^*\,[\mathbf{I}_{012}]^*$$

$$\mathbf{S}_{3\phi} = [\mathbf{V}_{abc}]^t\,[\mathbf{I}_{abc}]^*$$

$$\mathbf{S}_{3\phi} = [\mathbf{V}_{012}]^t\,[\mathbf{A}]^t\,[\mathbf{A}]^*\,[\mathbf{I}_{012}]^*$$

$$[\mathbf{A}]^t[\mathbf{A}]^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{S}_{3\phi} = 3\left[\mathbf{V}_{012}\right]^t\left[\mathbf{I}_{012}\right]^* = 3\left[\mathbf{V}_{a0} \quad \mathbf{V}_{a1} \quad \mathbf{V}_{a2}\right]\begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix}^*$$

$$\mathbf{S}_{3\phi} = 3\left[\mathbf{V}_{a0}\mathbf{I}_{a0}^* + \mathbf{V}_{a1}\mathbf{I}_{a1}^* + \mathbf{V}_{a2}\mathbf{I}_{a2}^*\right]$$

EXAMPLE 5.2

Assume that the phase voltages and currents of a three-phase system are given as

$$[\mathbf{V}_{abc}] = \begin{bmatrix} 0 \\ 50 \\ -50 \end{bmatrix} \quad \text{and} \quad [\mathbf{I}_{abc}] = \begin{bmatrix} -5 \\ j5 \\ -5 \end{bmatrix}$$

and determine the following:

(a) Three-phase complex power using Equation 5.30
(b) Sequence voltage and current matrices, that is, $[\mathbf{V}_{012}]$ and $[\mathbf{I}_{012}]$
(c) Three-phase complex power using Equation 5.34

$$\mathbf{S}_{3\phi} = \left[\mathbf{V}_{abc}\right]^{t}\left[\mathbf{I}_{abc}\right]^{*}$$

$$= \begin{bmatrix} 0 & 50 & -50 \end{bmatrix}\begin{bmatrix} 5 \\ -j5 \\ +5 \end{bmatrix}$$

$$= -250 - j250$$

$$= 353.5534\angle 45^{\circ}\ \text{VA}$$

$$\left[\mathbf{V}_{012}\right] = [\mathbf{A}]^{-1}\left[\mathbf{V}_{abc}\right]$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}\begin{bmatrix} 0 \\ 50 \\ -50 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0\angle 0^{\circ} \\ 28.8675\angle 90^{\circ} \\ 28.8675\angle -90^{\circ} \end{bmatrix}\ \text{V}$$

$$\left[\mathbf{I}_{012}\right] = [\mathbf{A}]^{-1}\left[\mathbf{I}_{abc}\right]$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}\begin{bmatrix} -5 \\ j5 \\ -5 \end{bmatrix}$$

$$= \begin{bmatrix} 3.7268\angle 153.4^{\circ} \\ 2.3570\angle 165^{\circ} \\ 2.3570\angle -75^{\circ} \end{bmatrix}$$

$$\mathbf{S}_{3\phi} = 3\left[\mathbf{V}_{a0}\mathbf{I}_{a0}^{*} + \mathbf{V}_{a1}\mathbf{I}_{a1}^{*} + \mathbf{V}_{a2}\mathbf{I}_{a2}^{*}\right]$$

$$= 353.5534\angle -45^{\circ}\ \text{VA}$$

TUGAS 6

1. Determine the symmetrical components for the phase currents of $\mathbf{I}_a = 125\angle 20°$, $\mathbf{I}_b = 175\angle -100°$, and, $\mathbf{I}_c = 95\angle 155°$ A.
2. Assume that the unbalanced phase currents are $\mathbf{I}_\mathrm{a} = 100\angle 180°$, $\mathbf{I}_\mathrm{b} = 100\angle 0°$, and, $\mathbf{I}_\mathrm{c} = 10\angle 20°$ A.
   (a) Determine the symmetrical components.
   (b) Draw a phasor diagram showing $\mathbf{I}_{a0}$, $\mathbf{I}_{a1}$, $\mathbf{I}_{a2}$, $\mathbf{I}_{b0}$, $\mathbf{I}_{b1}$, $\mathbf{I}_{b2}$, $\mathbf{I}_{c0}$, $\mathbf{I}_{c1}$, and $\mathbf{1}_{c2}$ (i.e., the positive-, negative-, and zero-sequence currents for each phase).
3. Assume that $\mathbf{V}_{a1} = 180\angle 0°$, $\mathbf{V}_{a2} = 100\angle 100°$, and, $\mathbf{V}_{a0} = 250\angle -40°$ V.
   (a) Draw a phasor diagram showing all the nine symmetrical components.
   (b) Find the phase voltages $[\mathbf{V}_{abc}]$ using the equation

$$[\mathbf{V}_{abc}] = [\mathbf{A}]\,[\mathbf{V}_{012}]$$

   (c) Find the phase voltages $[\mathbf{V}_{abc}]$ graphically and check the results against the ones found in part b.