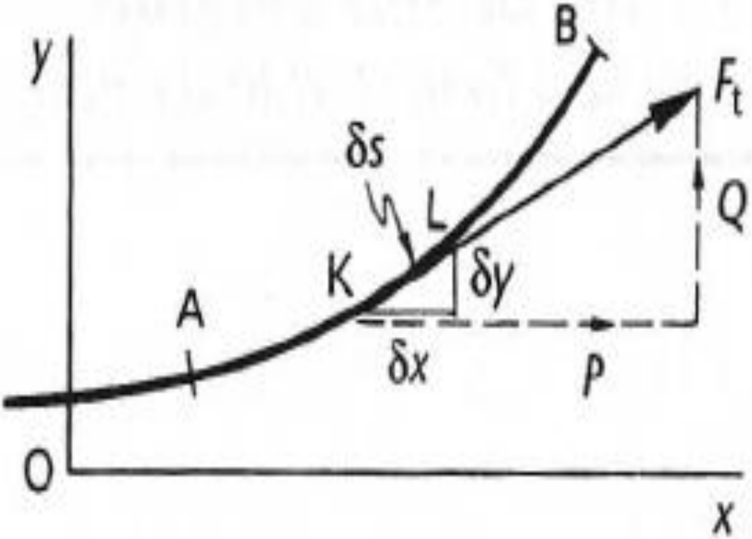
ITEGRAL GARIS

IR. I NYOMAN SETIAWAN
PS. TEKNIK ELEKTRO
E-mail :man_awan@yahoo.com
setiawan@.unud.ac.id

Integral Garis



If F_t has a component

P in the x-direction

Q in the y-direction

then the work done from K to L can be stated as $P\,\delta x + Q\,\delta y$.

$$\therefore \quad \int_{AB} F_t \, ds = \int_{AB} (P\,dx + Q\,dy)$$

where P and Q are functions of x and y.

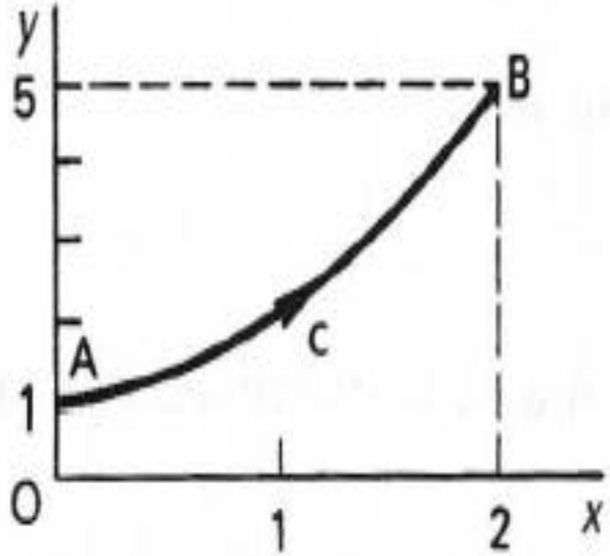
In general then, the line integral can be expressed as

$$I = \int_c F_t \, ds = \int_c (P\,dx + Q\,dy)$$

where c is the prescribed curve and F, or P and Q, are functions of x and y.

Contoh 1 :

Evaluate $\int_c (x+3y)\mathrm{d}x$ from A (0, 1) to B (2, 5) along the curve $y = 1 + x^2$.



The line integral is of the form

$$\int_c (P\,\mathrm{d}x + Q\,\mathrm{d}y)$$

where, in this case, $Q = 0$ and c is the curve $y = 1 + x^2$.

$$I = \int_c (P\,\mathrm{d}x + Q\,\mathrm{d}y) = \int_c (x+3y)\,\mathrm{d}x = \int_0^2 (x + 3 + 3x^2)\,\mathrm{d}x$$

$$= \left[\frac{x^2}{2} + 3x + x^3\right]_0^2 = 16$$

Contoh 2 :

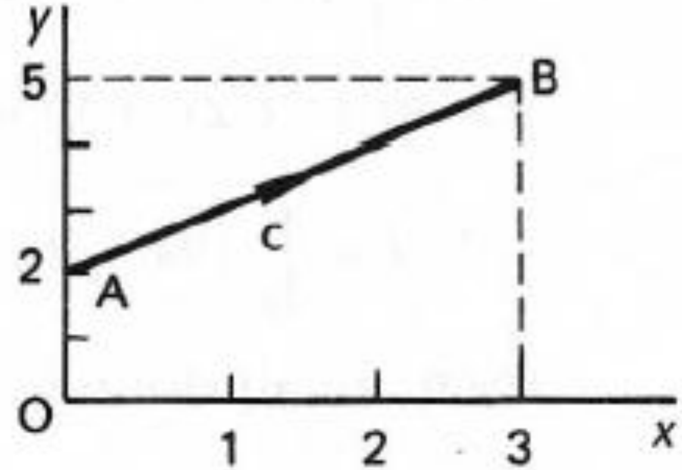
Evaluate $I = \int_c (x^2 + y)\,dx + (x - y^2)\,dy$ from A (0, 2) to B (3, 5) along the curve $y = 2 + x$.

$$I = \int_c (P\,dx + Q\,dy)$$

$$P = x^2 + y = x^2 + 2 + x = x^2 + x + 2$$

$$Q = x - y^2 = x - (4 + 4x + x^2)$$

$$= -(x^2 + 3x + 4)$$



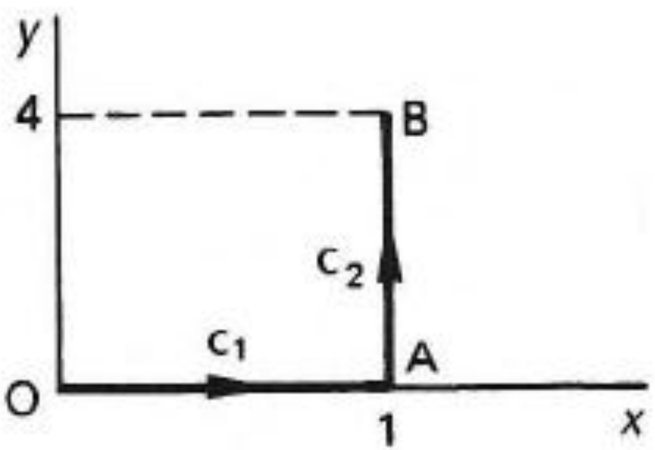
Also $y = 2 + x \quad \therefore \; dy = dx$ and the limits are $x = 0$ to $x = 3$.

$$I = \int_0^3 \{(x^2 + x + 2)\,dx - (x^2 + 3x + 4)\,dx\}$$

$$\int_0^3 -(2x + 2)dx = -\left[x^2 + 2x\right]_0^3 = -15$$

Contoh 3 :

Evaluate $I = \int_c \{(x^2 + 2y)\,dx + xy\,dy\}$ from O (0, 0) to A (1, 0) along line $y = 0$ and then from A (1, 0) to B (1, 4) along the line $x = 1$.



(1) OA: c_1 is the line $y = 0$ $\therefore$ $dy = 0$. Substituting $y = 0$ and $dy = 0$ in the given integral gives

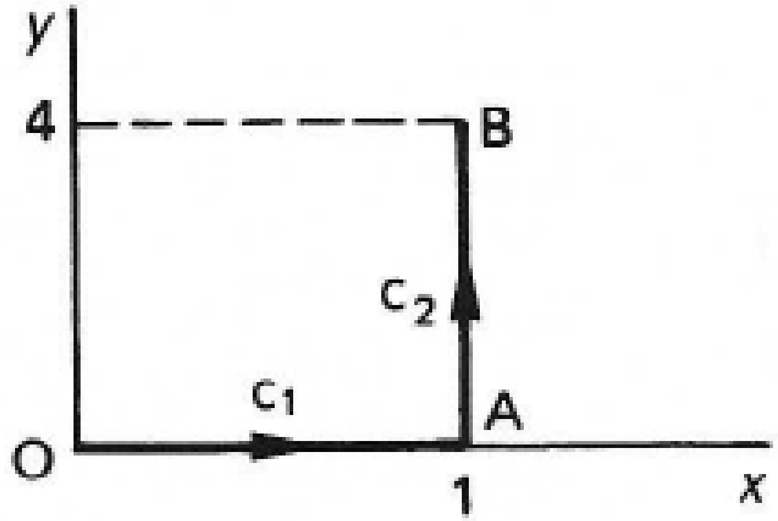
$$I_{OA} = \int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

(2) AB: Here c_2 is the line $x = 1$ $\quad \therefore \; dx = 0$

$$I_{AB} = \int_0^4 \{(1 + 2y)(0) + y\,dy\}$$

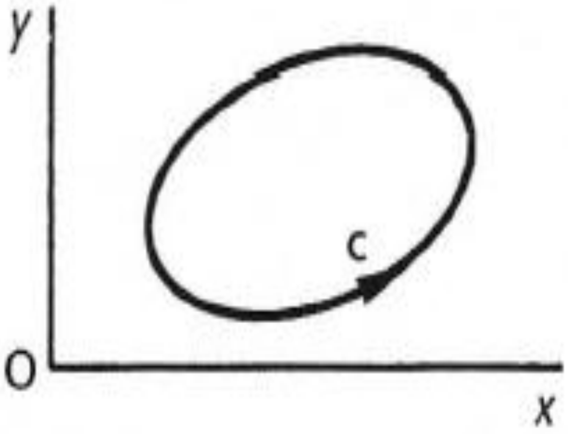
$$= \int_0^4 y\,dy$$

$$= \left[\frac{y^2}{2}\right]_0^4 = 8$$

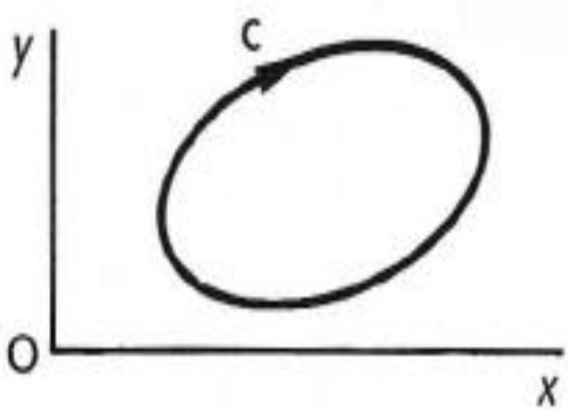


Then $I = I_{OA} + I_{AB} = \frac{1}{3} + 8 = 8\frac{1}{3}$ $\quad \therefore \; I = 8\frac{1}{3}$

Integral Garis pada lintasan kurva tertutup



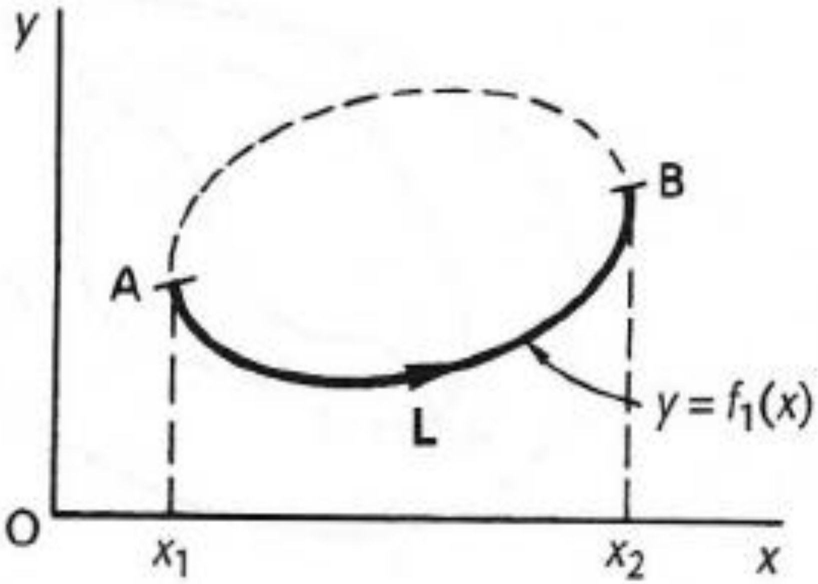
Positive direction (anticlockwise) line integral denoted by $\oint$.



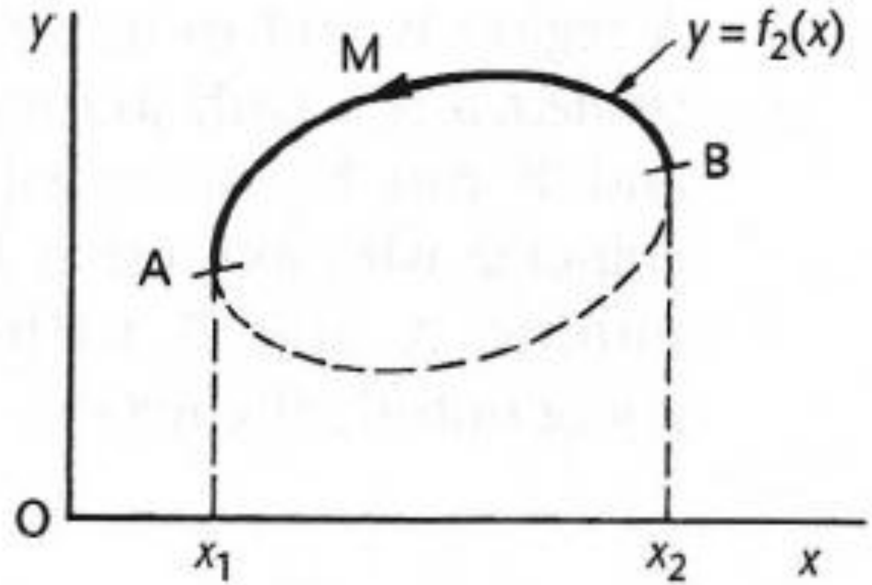
Negative direction (clockwise) line integral denoted by $-\oint$.

Integral Garis pada lintasan kurva tertutup

With a closed curve, the y-values on the path c cannot be single-valued. Therefore, we divide the path into two or more parts and treat each separately.



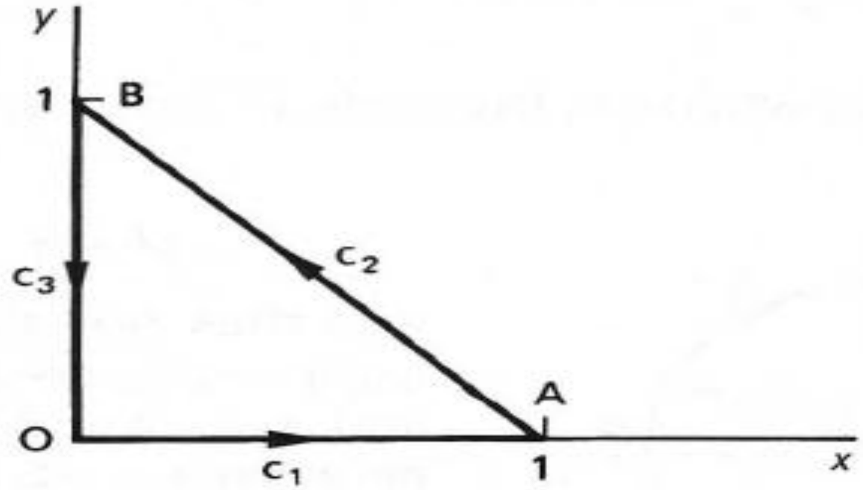
(1) Use $y = f_1(x)$ for ALB



(2) Use $y = f_2(x)$ for BMA.

Contoh 1 :

Evaluate the line integral $I = \oint_c (x^2\,dx - 2xy\,dy)$ where c comprises the three sides of the triangle joining O (0, 0), A (1, 0) and B (0, 1).



(a) OA: c_1 is the line $y = 0$ $\therefore$ $dy = 0$.

Then $I = \oint (x^2\,dx - 2xy\,dy)$ for this part becomes

$$I_1 = \int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3} \qquad \therefore \; I_1 = \frac{1}{3}$$

(b) AB: c_2 is the line $y = 1 - x \quad \therefore \; dy = -dx$

Because c_2 is the line $y = 1 - x \quad \therefore \; dy = -dx$.

$$I_2 = \int_1^0 \{x^2\,dx + 2x(1-x)\,dx\} = \int_1^0 (x^2 + 2x - 2x^2)\,dx$$

$$= \int_1^0 (2x - x^2)dx = \left[x^2 - \frac{x^3}{3}\right]_1^0 = -\frac{2}{3} \quad \therefore \; I_2 = -\frac{2}{3}$$

(c) BO: c_3 is the line $x = 0$

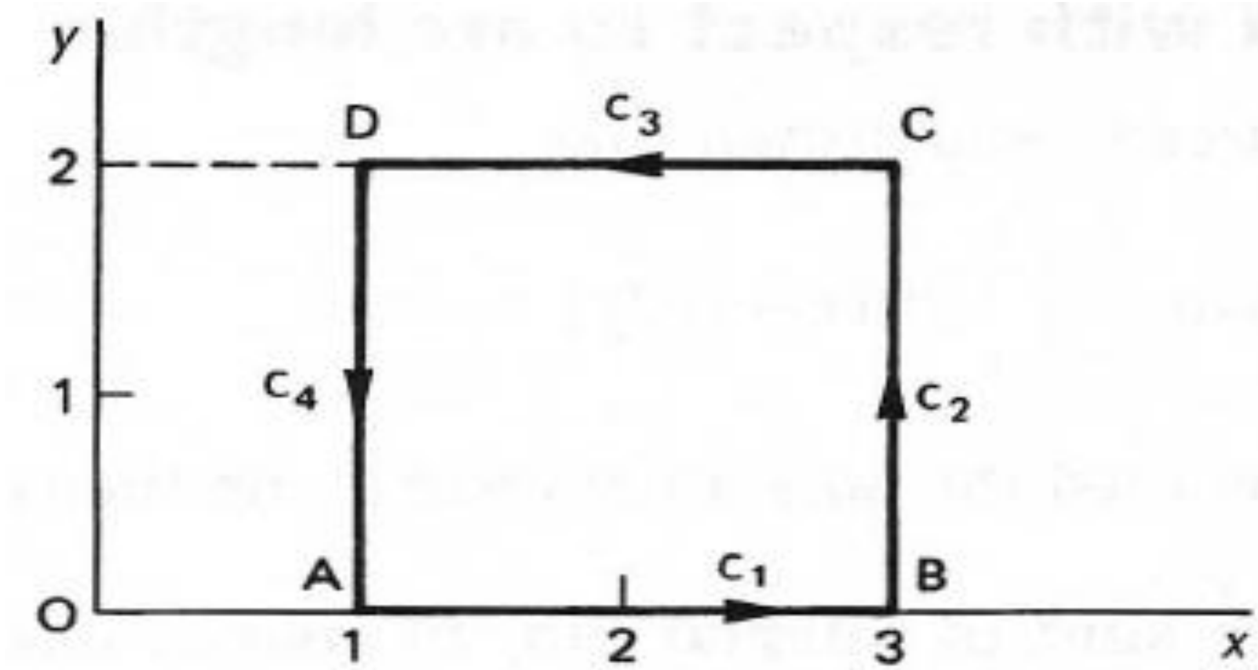
Because for c_3, $\quad x = 0 \quad \therefore \; dx = 0 \quad \therefore \; I_3 = \int 0\,dy = 0 \quad \therefore \; I_3 = 0$

Finally, $I = I_1 + I_2 + I_3 = \frac{1}{3} - \frac{2}{3} + 0 = -\frac{1}{3} \qquad \therefore \; I = -\frac{1}{3}$

Contoh 2 :

Evaluate $I = \oint_c \{xy\,dx + (1 + y^2)\,dy\}$ where c is the boundary of the rectangle joining A (1, 0), B (3, 0), C (3, 2) and D (1, 2).

First draw the diagram and insert c_1, c_2, c_3, c_4.



$$I = \oint_c \{xy\,dx + (1 + y^2)\,dy\}$$

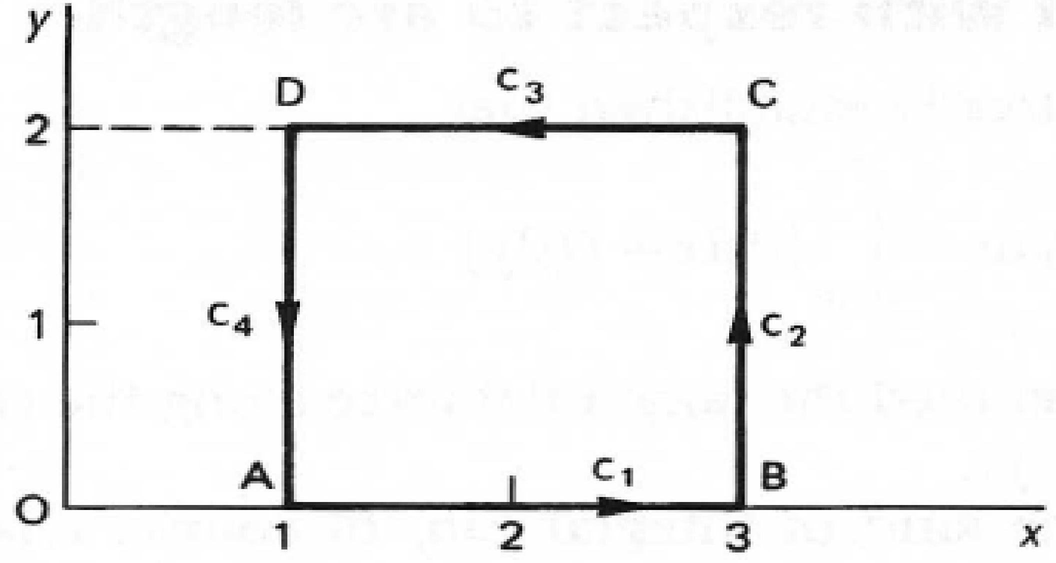
(a) AB: c_1 is $y = 0$ $\quad \therefore\ dy = 0 \quad \therefore\ I_1 = 0$

(b) BC: c_2 is $x = 3$ $\quad \therefore\ dx = 0$

$$\therefore\ I_2 = \int_0^2 (1 + y^2)dy = \left[y + \frac{y^3}{3}\right]_0^2 = 4\tfrac{2}{3} \qquad \therefore\ I_2 = 4\tfrac{2}{3}$$

(c) CD: c_3 is $y = 2$ $\quad \therefore\ dy = 0$

$$\therefore\ I_3 = \int_3^1 2x\,dx = \left[x^2\right]_3^1 = -8 \qquad \therefore\ I_3 = -8$$



(d) DA: c_4 is $x = 1$ $\quad \therefore \; dx = 0$

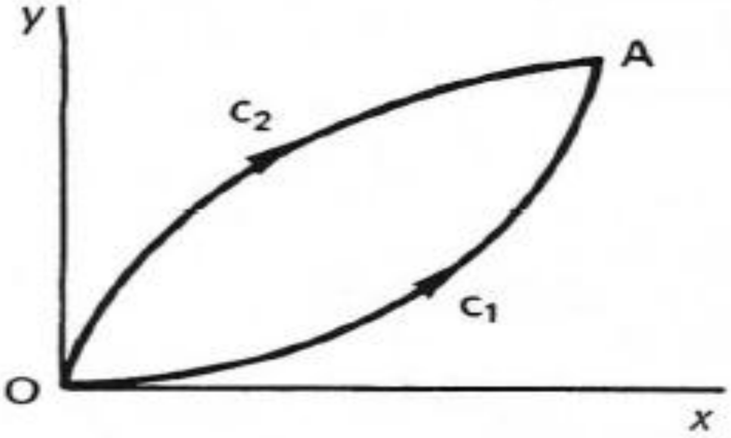
$$\therefore \; I_4 = \int_2^0 (1 + y^2)\,dy = \left[y + \frac{y^3}{3} \right]_2^0 = -4\tfrac{2}{3} \quad \therefore \; I_4 = -4\tfrac{2}{3}$$

$$I = I_1 + I_2 + I_3 + I_4$$

$$= 0 + 4\tfrac{2}{3} - 8 - 4\tfrac{2}{3} = -8 \qquad \therefore \; I = -8$$

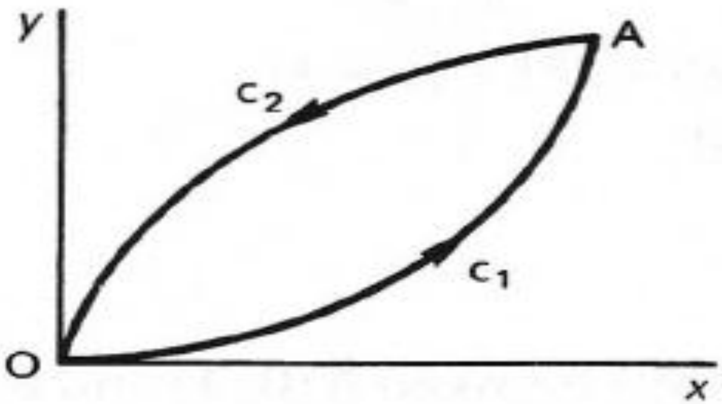
Diferensial Eksak

$$\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$$



If $I = \int_c \{P\,dx + Q\,dy\}$ and $(P\,dx + Q\,dy)$ is an exact differential, then

$$I_{c_1} = I_{c_2}$$



If we reverse the direction of c_2, then

$$I_{c_1} = -I_{c_2}$$

i.e. $I_{c_1} + I_{c_2} = 0$

Hence, *if* $(P\,dx + Q\,dy)$ *is an exact differential, then the integration taken round a closed curve is zero.*

$\therefore$ If $(P\,dx + Q\,dy)$ is an exact differential, $\oint(P\,dx + Q\,dy) = 0$