Perkalian dan Pembagian Bilangan Kompleks dalam bentuk Kutub

Perkalian

$$
\begin{aligned}
z_1 z_2 &= r_1(\cos\theta_1 + j\sin\theta_1) r_2(\cos\theta_2 + j\sin\theta_2) \\
&= r_1 r_2(\cos(\theta_1+\theta_2) + j\sin(\theta_1+\theta_2)) \\
&= r_1 r_2 \angle \theta_1 + \theta_2
\end{aligned}
$$

Pembagian

$$
\begin{aligned}
\frac{z_1}{z_2} &= \frac{r_1(\cos\theta_1 + j\sin\theta_1)}{r_2(\cos\theta_2 + j\sin\theta_2)} \\
&= \frac{r_1(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 - j\sin\theta_2)}{r_2(\cos\theta_2 + j\sin\theta_2)(\cos\theta_2 - j\sin\theta_2)}
\end{aligned}
$$

$$\frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 - j\sin\theta_2)}{r_2(\cos\theta_2 + j\sin\theta_2)(\cos\theta_2 - j\sin\theta_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1(\cos\theta_1\cos\theta_2 + j\sin\theta_1\cos\theta_2 - j\cos\theta_1\sin\theta_2 + \sin\theta_1\sin\theta_2)}{r_2(\cos^2\theta_2 + \sin^2\theta_2)}$$

$$= \frac{r_1(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + j(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)}{r_2(1)}$$

$$= \frac{r_1}{r_2}\left(\cos(\theta_1 - \theta_2) + j(\sin(\theta_1 - \theta_2))\right)$$

$$\boxed{\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle\theta_1 - \theta_2}$$

Contoh :

$$z_1 = 10(\cos 60° + j\sin 60°) = 10\angle 60°$$

$$z_2 = 5(\cos 40° + j\sin 40°) = 5\angle 40°$$

$$z_1 z_2 = 10(\cos 60° + j\sin 60°)5(\cos 40° + j\sin 40°)$$

$$= 50(\cos 100° + j\sin 100°) = 50\angle 100°$$

$$\frac{z_1}{z_2} = \frac{10(\cos 60° + j\sin 60°)}{5(\cos 40° + j\sin 40°)}$$

$$= 2(\cos 20° + j\sin 20°) = 2\angle 20°$$

Perpangkatan Bilangan Kompleks Bentuk Kutub

$$z_1 = r(\cos\theta + j\sin\theta) = r\angle\theta$$

$$z_2 = r(\cos\theta + j\sin\theta) = r\angle\theta$$

$$z_2 = z_1$$

$$z_1 z_1 = r(os\theta + j\sin\theta) r(\cos\theta + j\sin\theta)$$

$$= r.r(\cos(\theta+\theta) + j\sin(\theta+\theta))$$

$$z_1^2 = r^2\angle 2\theta$$

Teorema DeMoivre (*ABRAHAM DE MOIVRE(1667-1754)*)

$$z^n = r^n(\cos n\theta + j\sin n\theta) = r^n\angle n\theta$$

Penjabaran cos $n\theta$ dan sin $n\theta$

$$\cos 3\theta + j\sin 3\theta = (\cos\theta + j\sin\theta)^3$$

$$\cos 3\theta + j\sin 3\theta = \cos^3\theta + 3\cos^2\theta(j\sin\theta) + 3\cos\theta(j\sin\theta)^2 + (j\sin\theta)^3$$

$$= \cos^3\theta + j3\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - j\sin^3\theta$$

$$= (\cos^3\theta - 3\cos\theta\sin^2\theta) + j(3\cos^2\theta\sin\theta - \sin^3\theta)$$

riil

imajiner

$$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$$

$$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$$

Samakan bagian riil dan bagian imajiner

Jika $\sin^2\theta = (1-\cos^2\theta)$ *maka* $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Jika $\cos^2\theta = \left(1-\sin^2\theta\right)$ *maka* $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

Penjabaran $\cos^n\theta$ dan $\sin^n\theta$ *dinyatakan dalam sinus dan cosinus kalipatan* θ

$$z^n = \cos n\theta + j\sin n\theta$$

$$\frac{1}{z^n} = z^{-n} = \cos n\theta - j\sin n\theta$$

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

$$z^n - \frac{1}{z^n} = j2\sin n\theta$$

Contoh : jabarkan $\cos^3\theta$

$$z + \frac{1}{z} = 2\cos\theta$$

$$(2\cos\theta)^3 = \left(z + \frac{1}{z}\right)^3 = z^3 + 3z^2\left(\frac{1}{z}\right) + 3z\left(\frac{1}{z^2}\right) + \frac{1}{z^3}$$

$$= \left(z^3 + \frac{1}{z^3}\right) + 3\left(z + \frac{1}{z}\right)$$

$$(2\cos\theta)^3 = 2\cos 3\theta + 3(2\cos\theta)$$

$$8\cos^3\theta = 2\cos 3\theta + 6\cos\theta$$

$$4\cos^3\theta = \cos 3\theta + 3\cos\theta$$

$$\cos^3\theta = \frac{1}{4}(\cos 3\theta + 3\cos\theta)$$

Tempat Kududukan

Contoh :

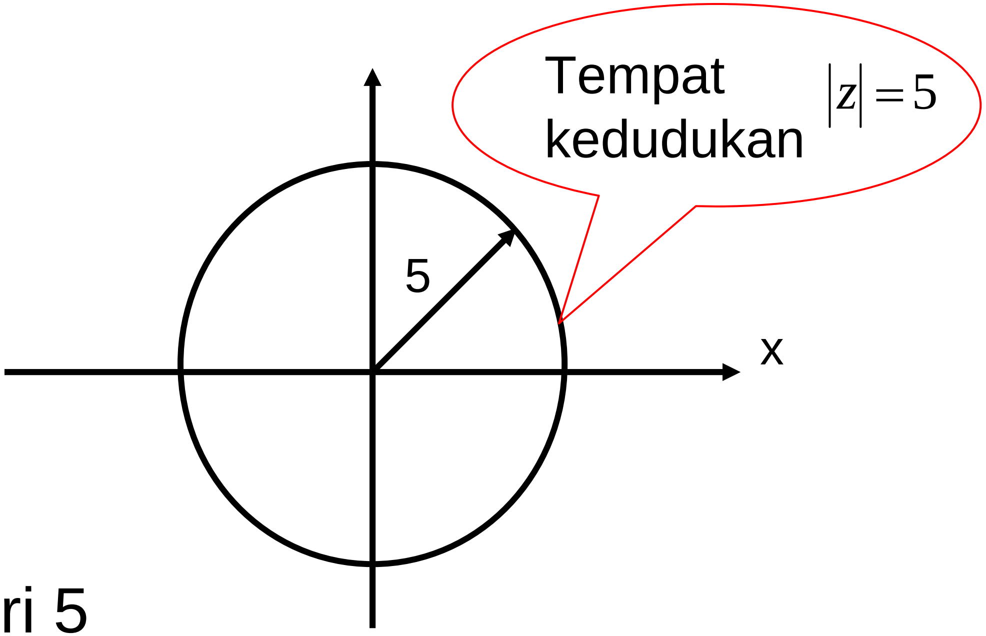
Jika $z = x + jy$, tentukan tempat kedudukan yang didefinisikan oleh $|z| = 5$

$$|z| = \sqrt{x^2 + y^2} = 5$$

$$\sqrt{x^2 + y^2} = 5$$

$$x^2 + y^2 = 25$$

Persamaan lingkaran yang berpusat (0,0) dan berjari-jari 5



Contoh :

$$\text{Jika } z = x + jy, \text{ tentukan tempat kedudukan}$$

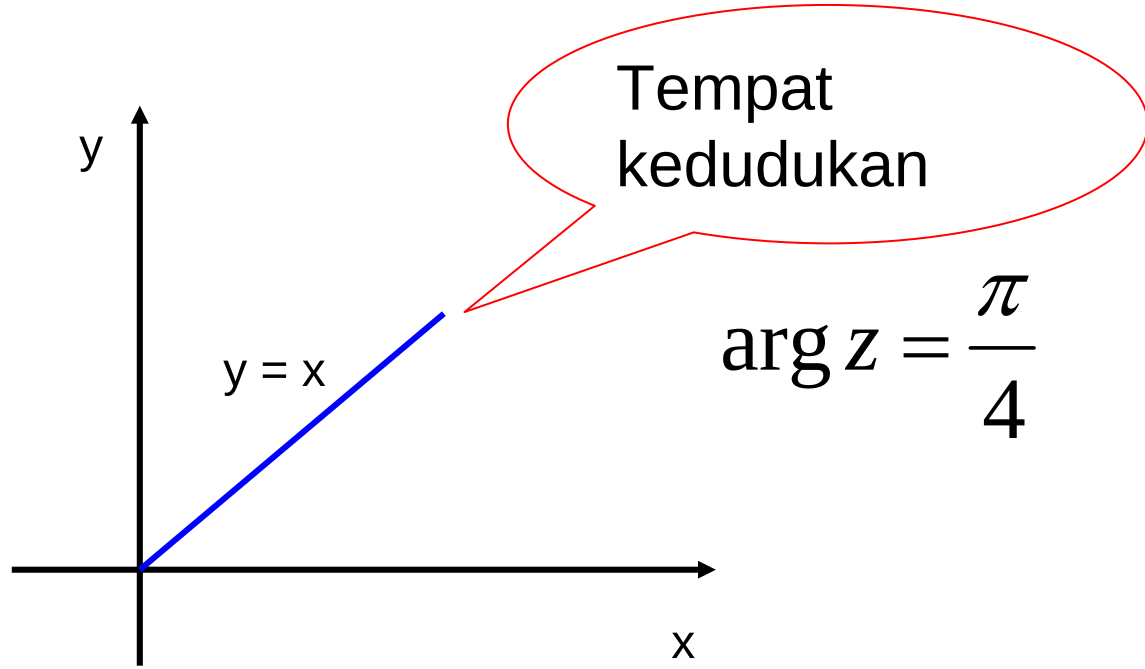
$$\text{yang didefinisikan oleh } \arg z = \frac{\pi}{4}$$

$$\arg z = \tan^{-1}\frac{y}{x}$$

$$\tan^{-1}\frac{y}{x} = \frac{\pi}{4}$$

$$\frac{y}{x} = \tan\frac{\pi}{4} = \tan 45^\circ = 1$$

$$\frac{y}{x} = 1, \qquad \Rightarrow y = x$$

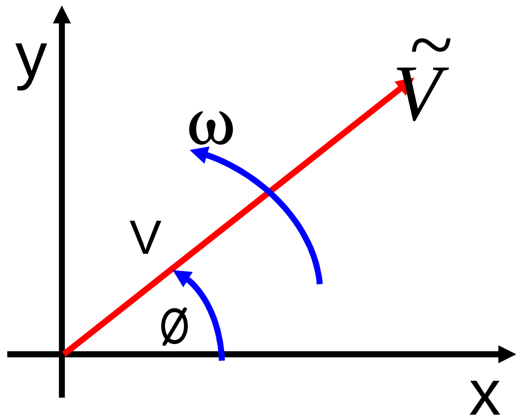


$$\arg z = \frac{\pi}{4}$$

$$\text{Jadi tempat kedudukannya adalah garis lurus } y = x \text{ dengan } y > 0$$

Phasor

$$v(t) = V_m \cos(\omega t + \phi) = V_m \cos(2\pi f t + \phi)$$



$$V_m = peak \ value$$

$$\omega = angular \ \ frequency$$

$$\phi = phase \ \ angle$$

Phasor tegangan dan arus

$$\tilde{V} = V\angle\phi$$

$$\tilde{I} = I\angle\phi$$

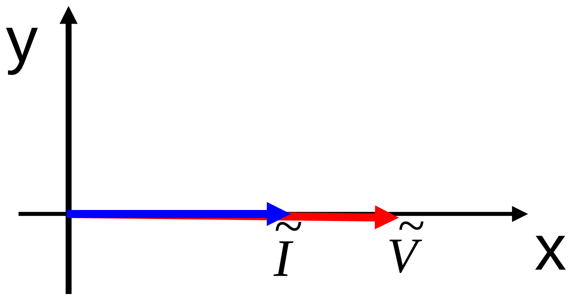
$$\tilde{V} = \tilde{I}Z$$

Phasor Resistor, Induktor dan Kapasitor

Tahanan (Resistor)

$$\tilde{I} = I\angle 0, \qquad Z = R\angle 0$$

$$\tilde{V} = IR\angle 0$$



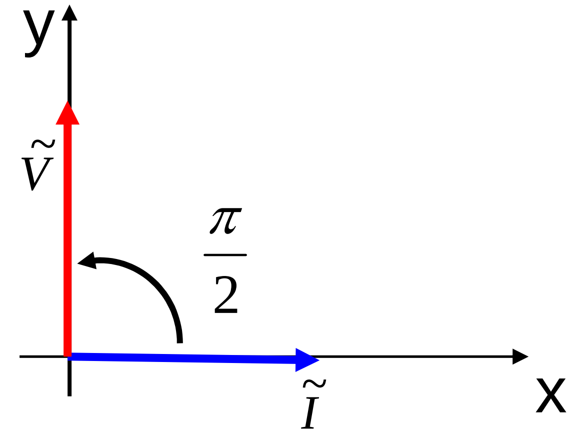
Induktor

$$\tilde{I} = I\angle 0, \qquad Z = \omega L\angle \pi/2$$

$$\tilde{V} = I\omega L\angle \pi/2$$

$$Z = \omega L e^{j\pi/2} = \omega L\left(\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}\right)$$

$$= j\omega L$$



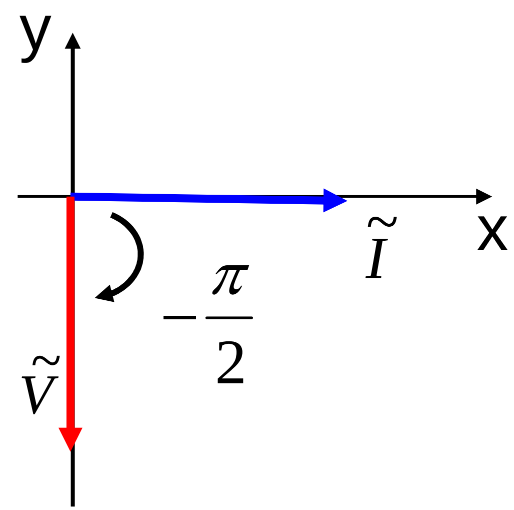
Kapasitor

$$\tilde{I} = I\angle 0, \qquad Z = \frac{1}{\omega C}\angle -\pi/2$$

$$\tilde{V} = \frac{I}{\omega C}\angle -\pi/2$$

$$Z = \frac{e^{-j\pi/2}}{\omega C} = \frac{1}{\omega C}\left(\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right)$$

$$= -\frac{j}{\omega C} \qquad \text{atau} \qquad Z = \frac{1}{j\omega C}$$



Rangkaian RLC

Hukum Kirchhoff

$$\tilde{V}_S = \tilde{V}_R + \tilde{V}_C + \tilde{V}_L$$

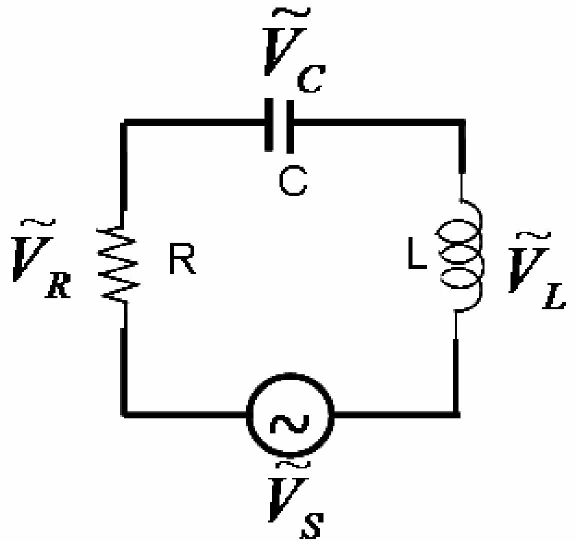
$$\tilde{V}_R = \tilde{I}R\angle 0 = \tilde{I}R$$

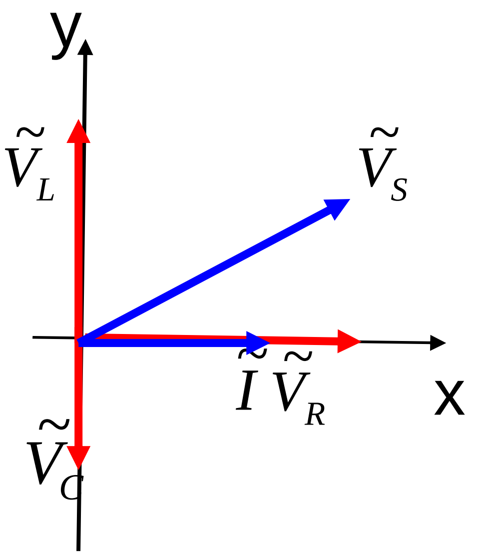
$$\tilde{V}_L = \tilde{I}\,\omega L\angle \pi/2 = j\tilde{I}\,\omega L$$

$$\tilde{V}_C = \frac{\tilde{I}}{\omega C}\angle -\pi/2 = \frac{\tilde{I}}{j\omega C}$$

$$\tilde{V}_S = \tilde{I}R + \tilde{I}j\omega L + \frac{\tilde{I}}{j\omega C}$$

$$\tilde{V}_S = \tilde{I}\left(R + j\omega L + \frac{1}{j\omega C}\right)$$





$$Z = R + j\omega L + \frac{1}{j\omega C}$$

Problem 8. A circuit comprises a resistance of 90 Ω in series with an inductor of inductive reactance 150 Ω. If the supply current is $1.35\angle 0^\circ$ A, determine (a) the supply voltage, (b) the voltage across the 90 Ω resistance, (c) the voltage across the inductance, and (d) the circuit phase angle. Draw the phasor diagram.

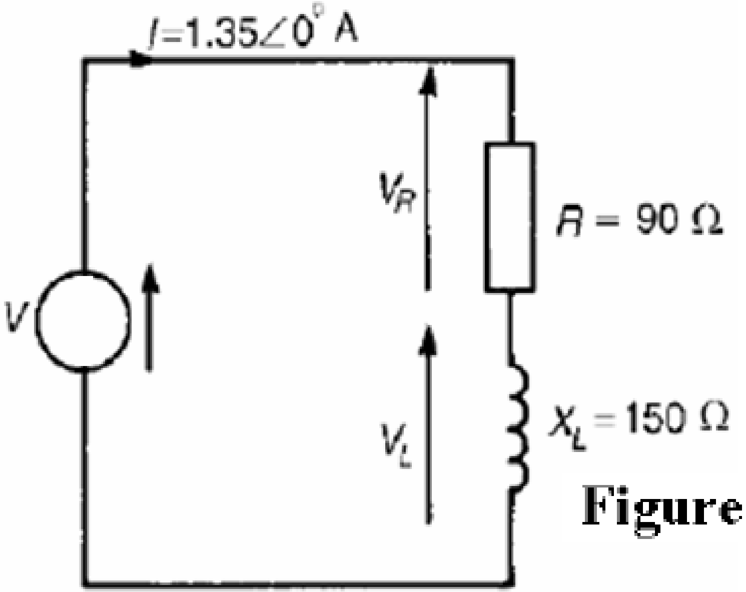


Figure 24.12

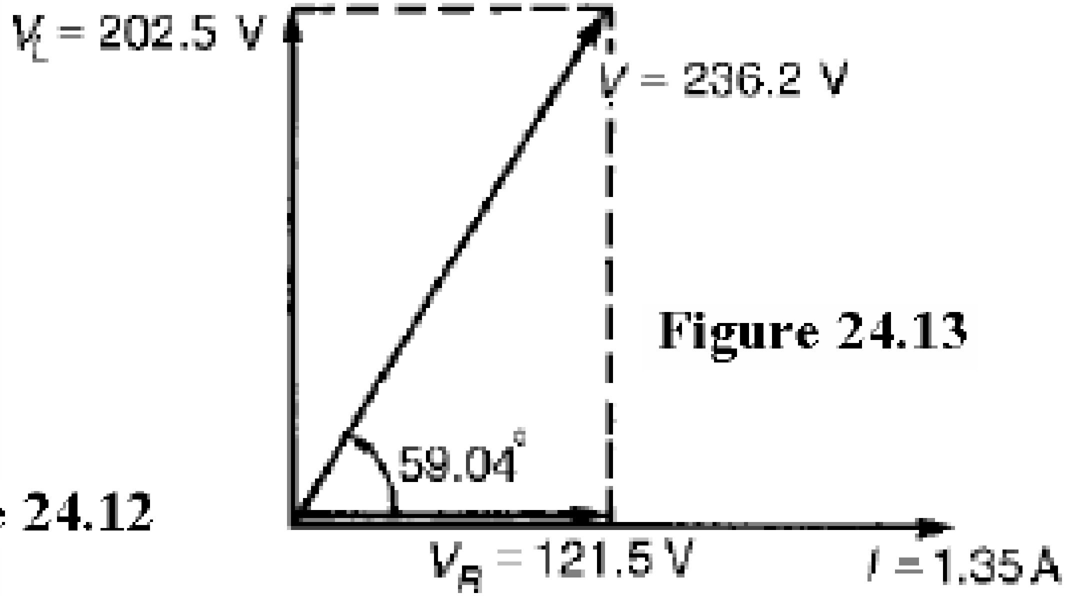


Figure 24.13

The circuit diagram is shown in Figure 24.12

(a) Circuit impedance $Z = R + jX_L = (90 + j150)\Omega$ or $174.93\angle 59.04^\circ\ \Omega$

Supply voltage, $V = IZ = (1.35\angle 0^\circ)(174.93\angle 59.04^\circ)$

$= \mathbf{236.2\angle 59.04^\circ\ V}$ **or** $\mathbf{(121.5 + j202.5)V}$

(b) Voltage across 90 Ω resistor, $V_R =$ **121.5 V** (since $V = V_R + jV_L$)

(c) Voltage across inductance, $V_L =$ **202.5 V** leading V_R by 90°.

(d) Circuit phase angle is the angle between the supply current and voltage, i.e., **59.04°** lagging(i.e., current lags voltage). The phasor diagram is shown in Figure 24.13.